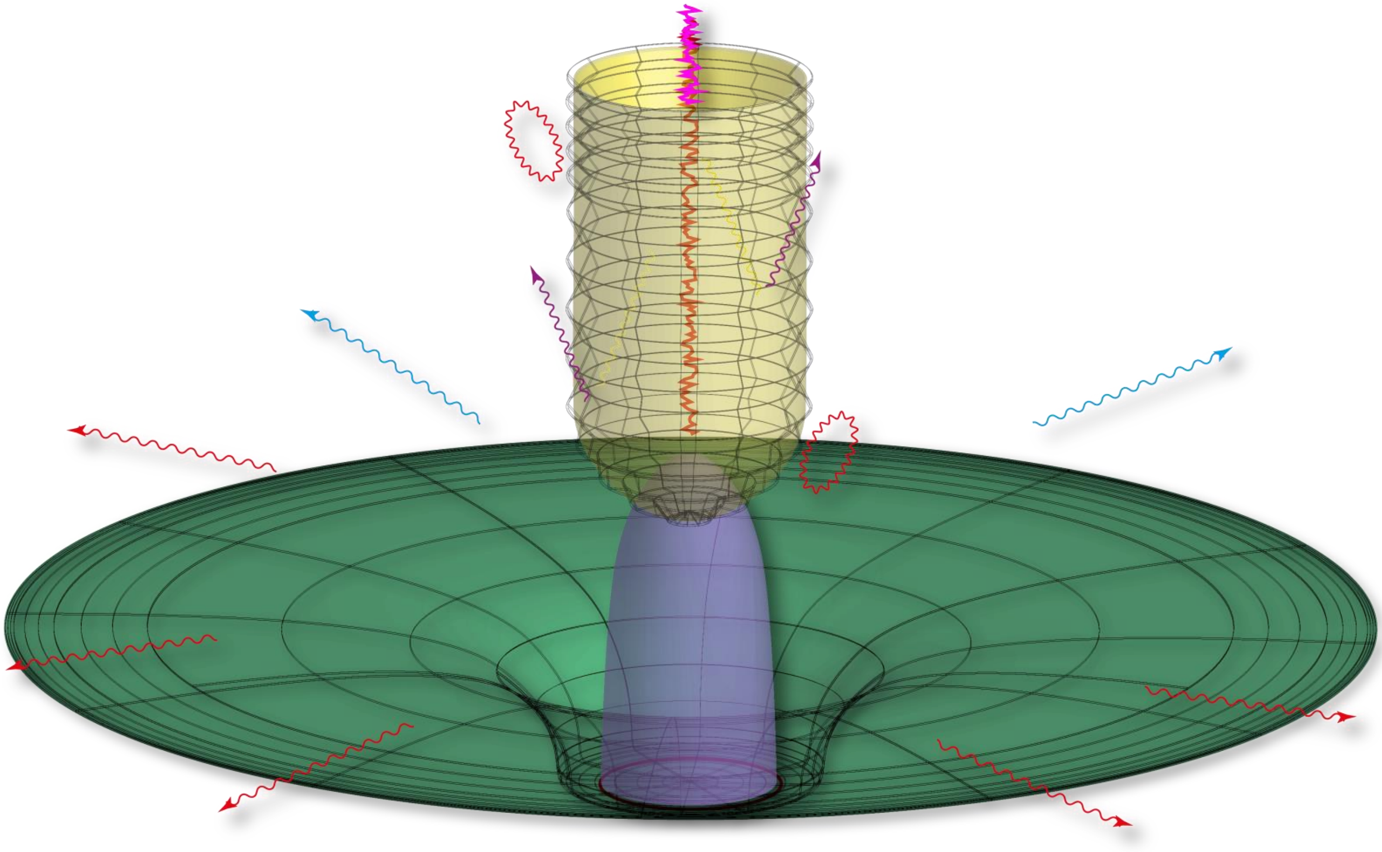
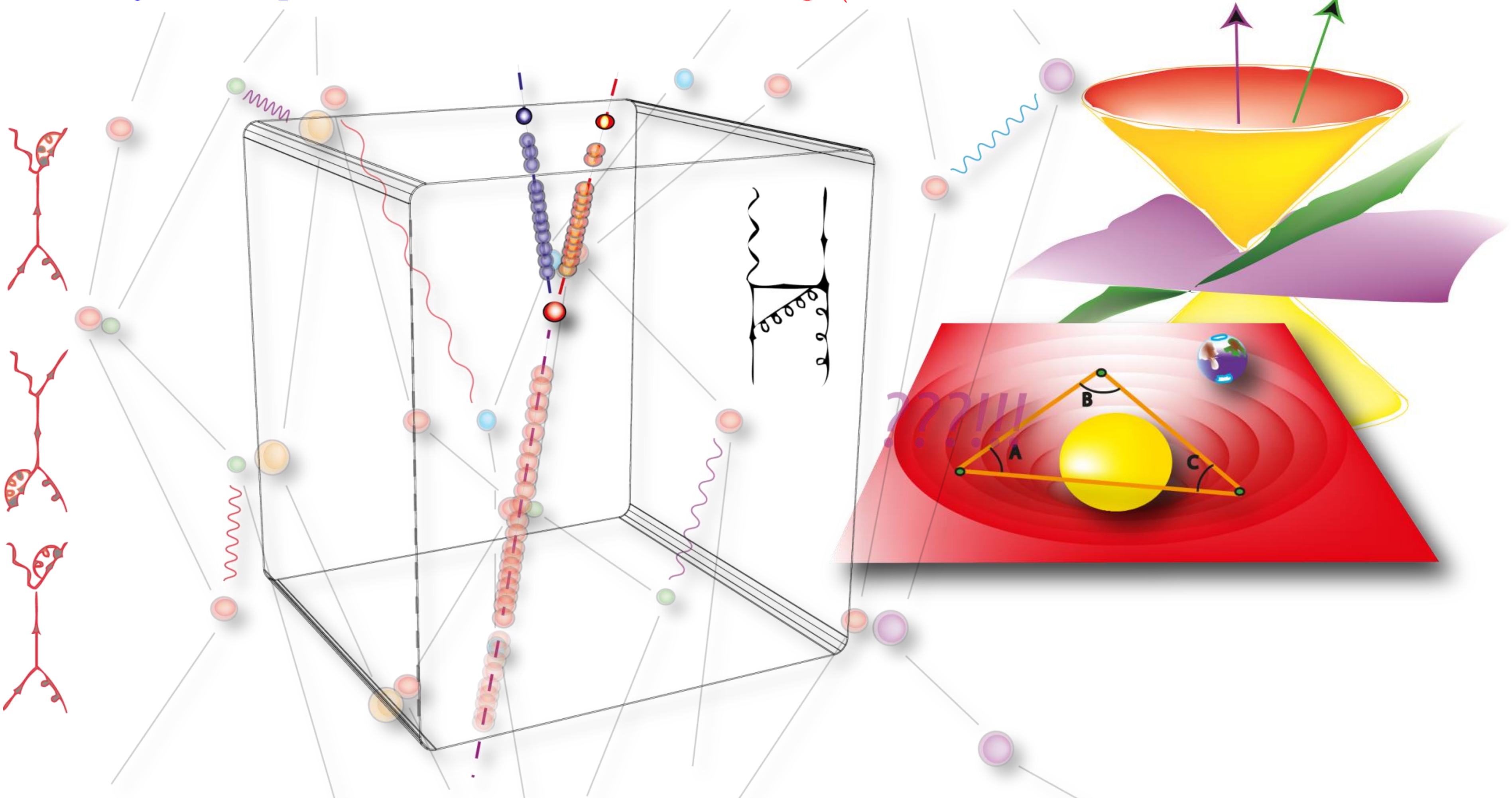
# Hawking



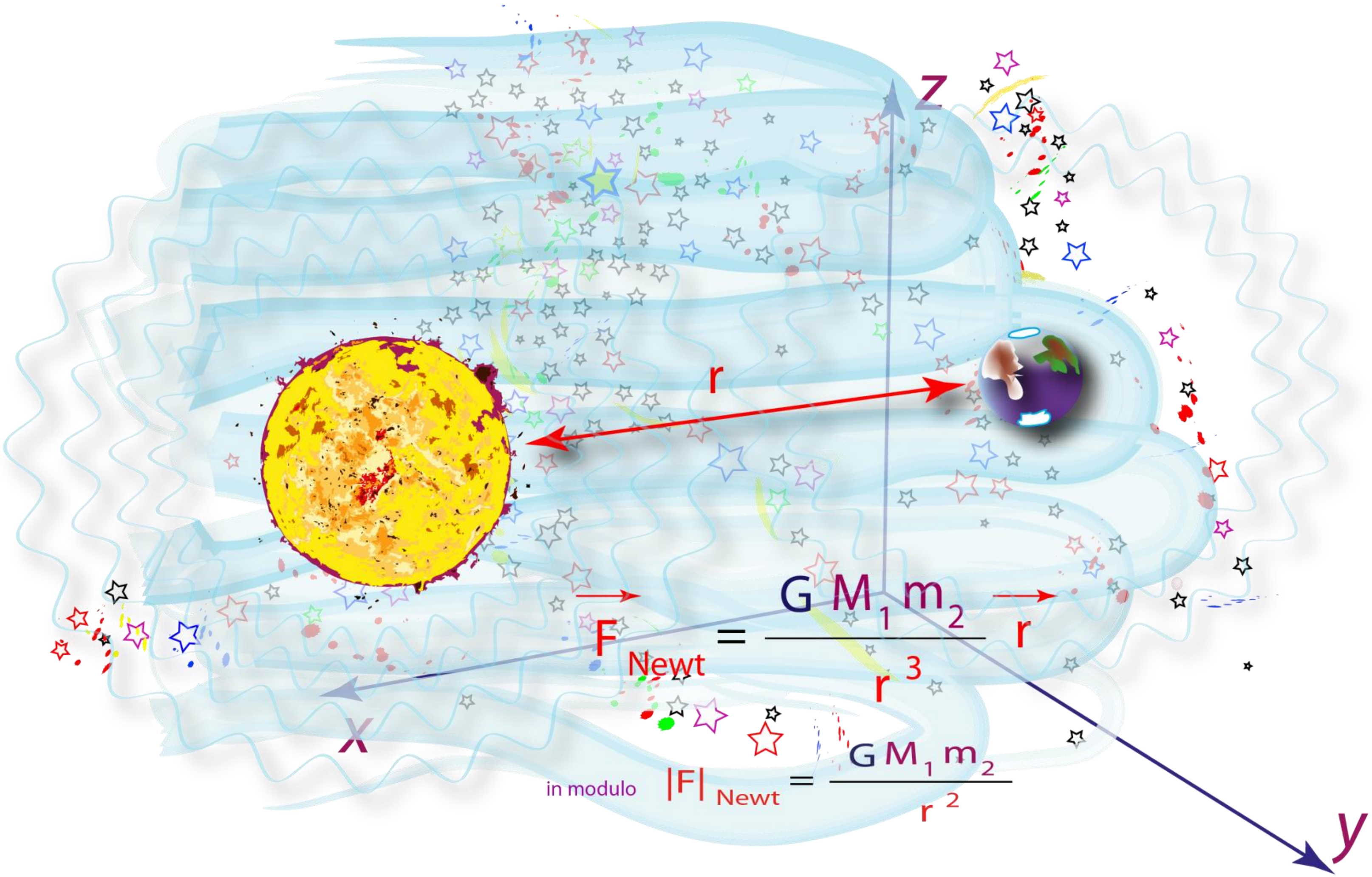
Mauro Carfora, Università di Pavia,
Dipartimento di Fisica, 8 Maggio 2018

The story of Hawking's scientific contributions starts in classical General Relativity, and follows a path that eventually led him to establish what is arguably the most significant result in the still ongoing attempt to reconcile general relativity with quantum mechanics: *Hawking (Black Hole) Radiation*.



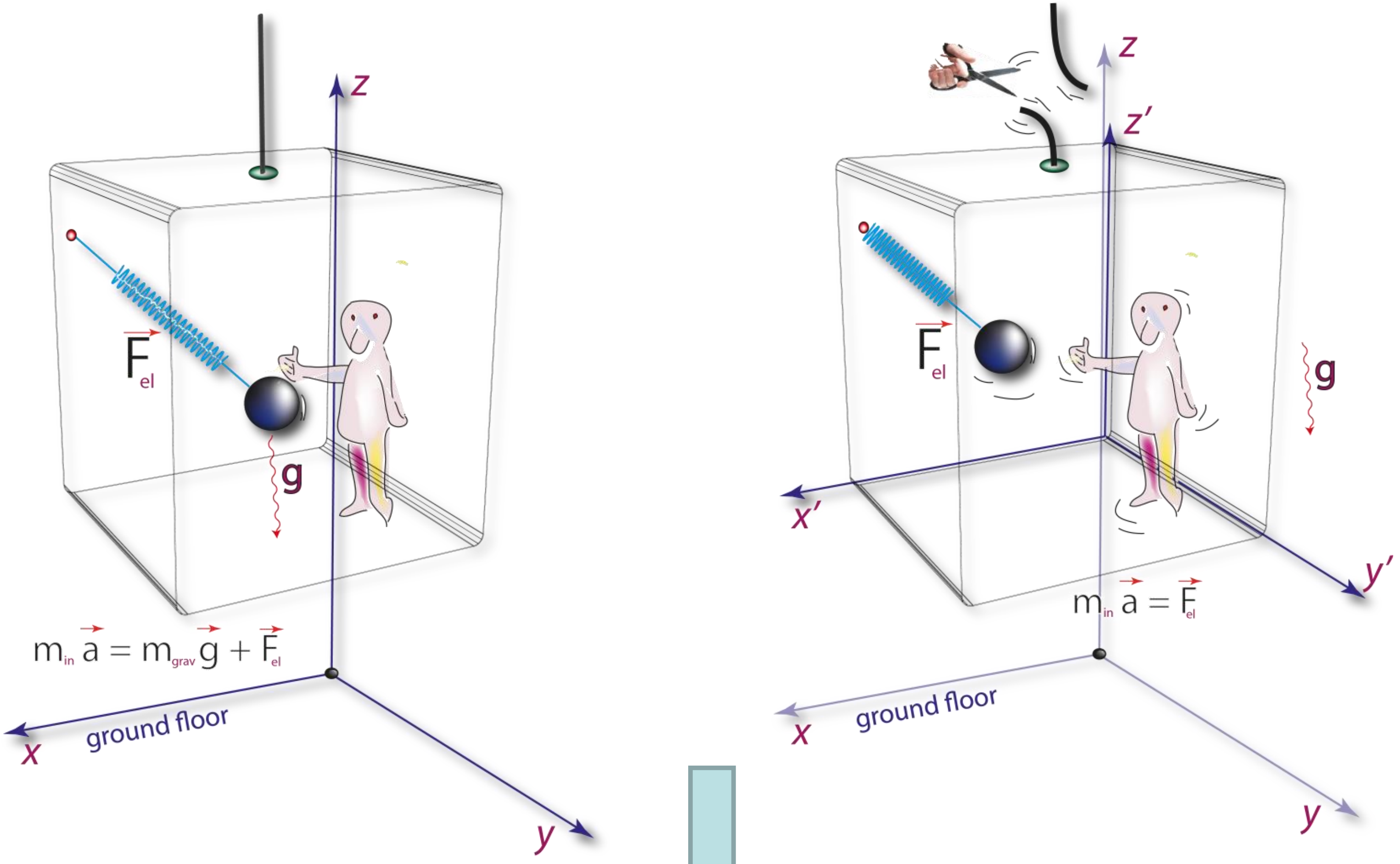
An attempt which represents *the most fundamental unsolved foundational issue in Physics* (R. Penrose)

...Hence, the story begins with... Gravity... the weakest among the known interactions, but the most evident because it is universal: gravity acts in the same manner on every form of mass–energy, hence its effects are cumulative.



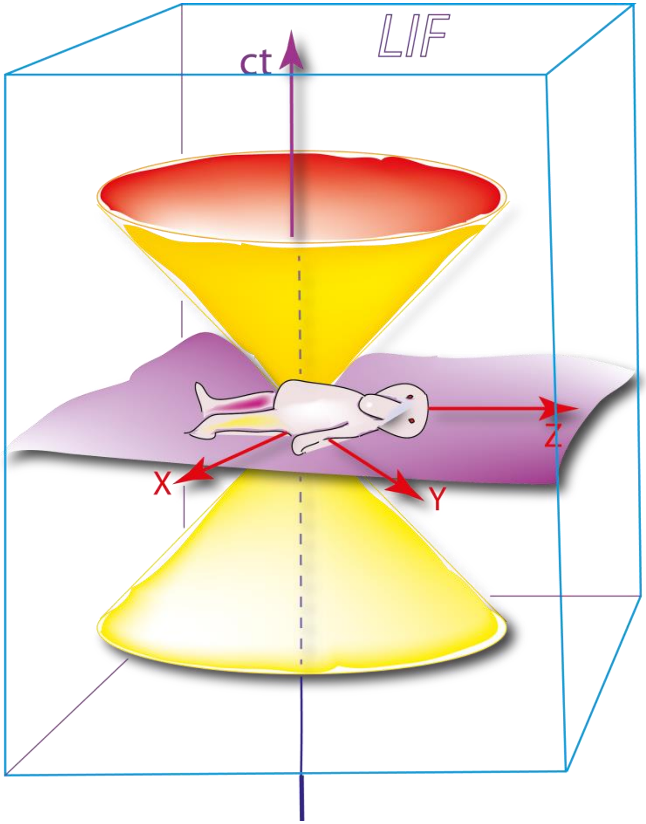
Gravity is a force which affects every particle in the same way, as first recognized by Galileo.

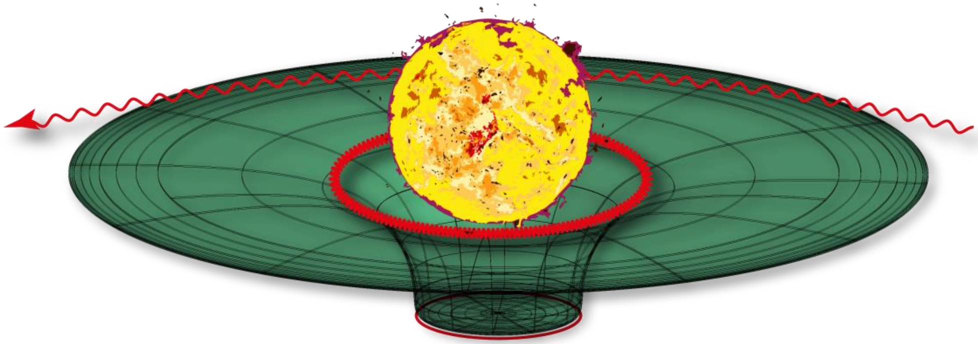
This implies that the gravitational field is locally eliminable.



In a Local Inertial Frame (**LIF**) we experience the Physics as in Minkowski spacetime (Special Relativity): (Einstein's equivalence principle).

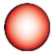
In a Local Inertial Frame (**LIF**) light propagates in vacuum as in special relativity. Its propagation is described by the collection of **local light cones** associated with the LIFs surrounding the gravitational source: Gravity bends light (both via the geometry of time and, in a more significant way, via the geometry of space)

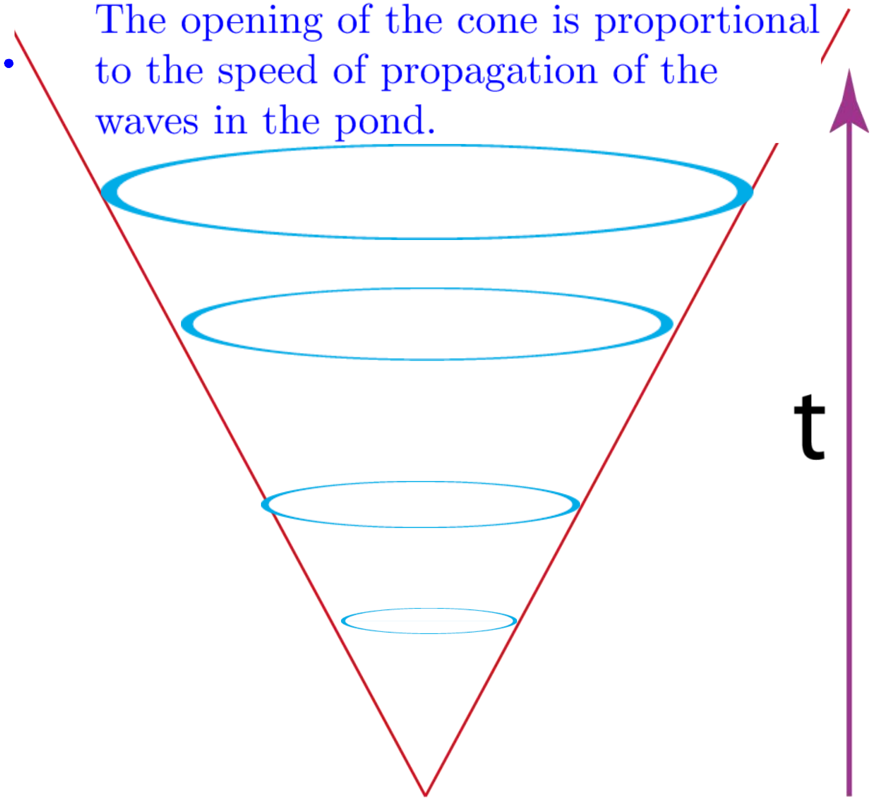




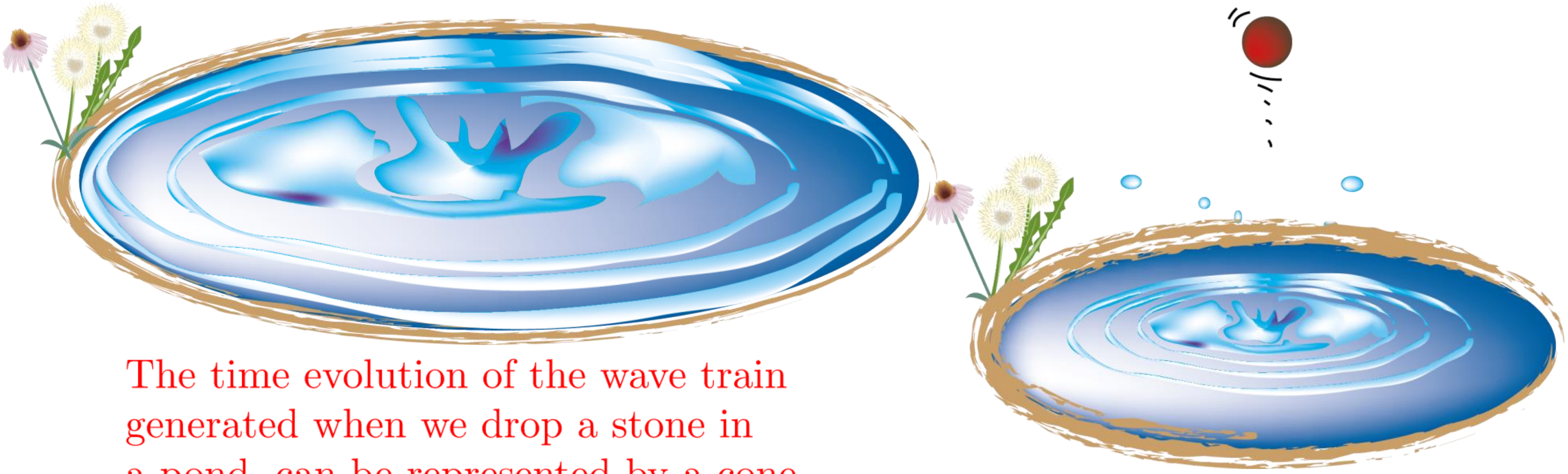
The basic role of the **Light Cone and causality** in general relativity:
**the key for understanding Hawking's results**

# … Light Cone heuristics…



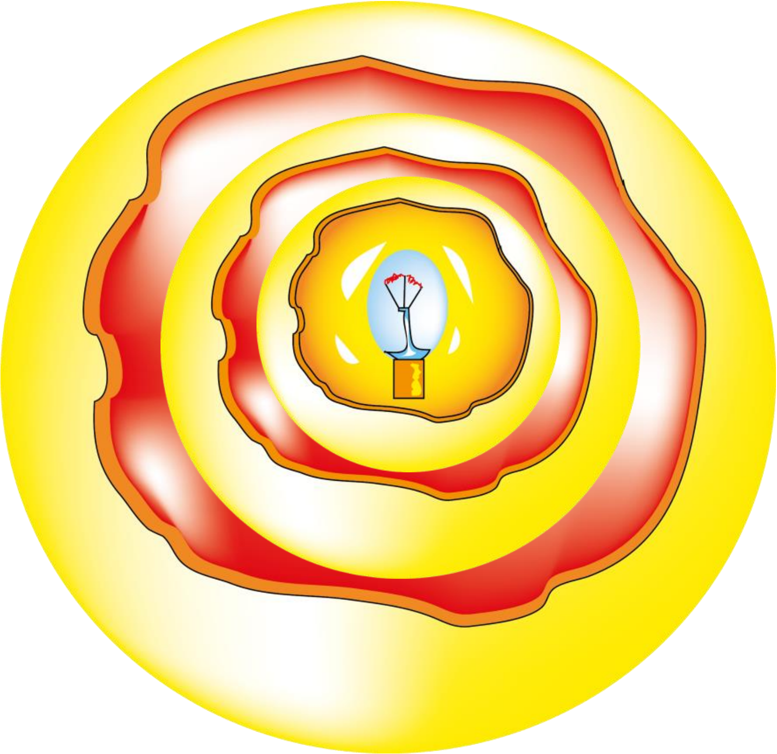


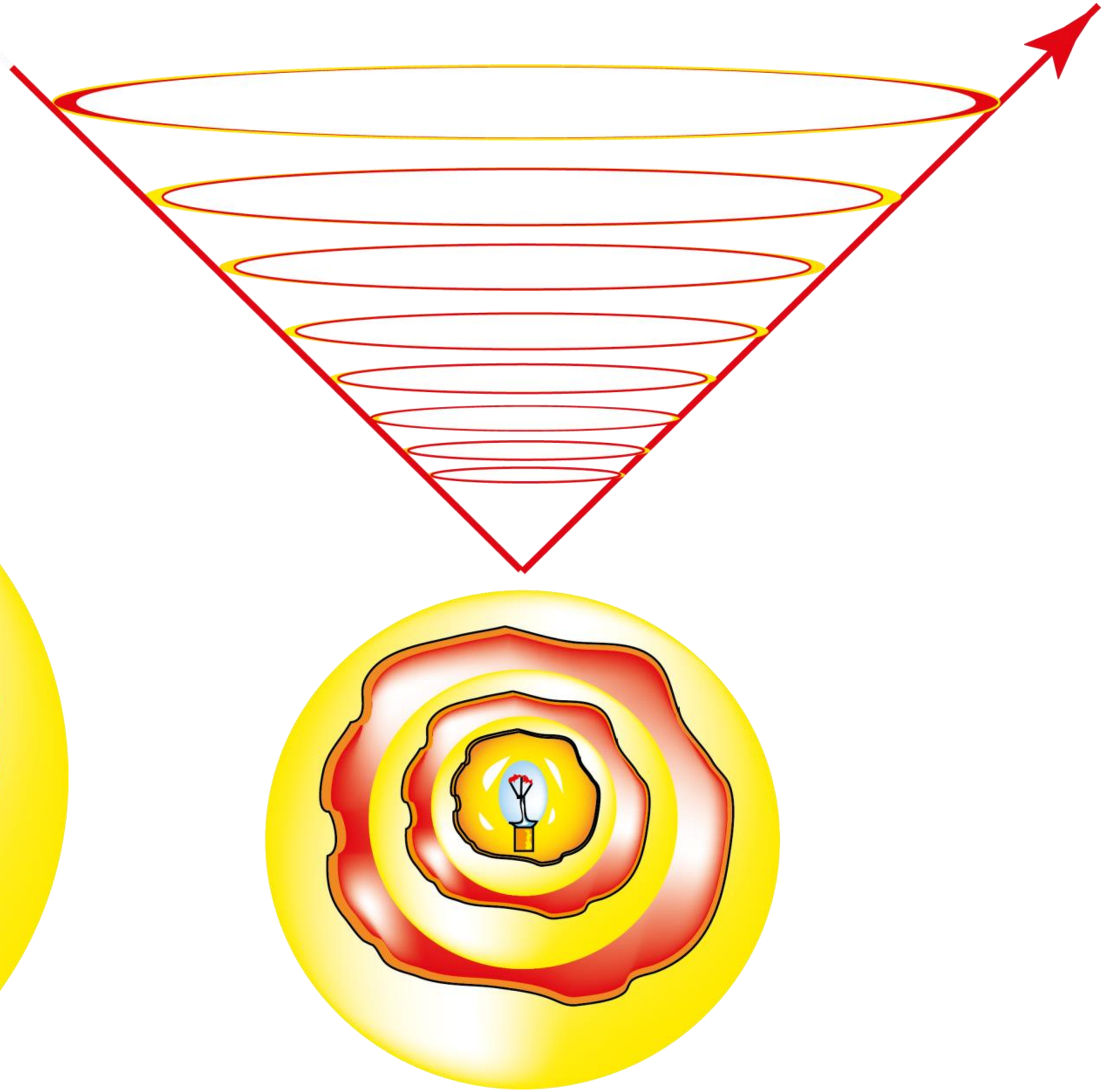
The opening of the cone is proportional to the speed of propagation of the waves in the pond.



The time evolution of the wave train generated when we drop a stone in a pond, can be represented by a cone.

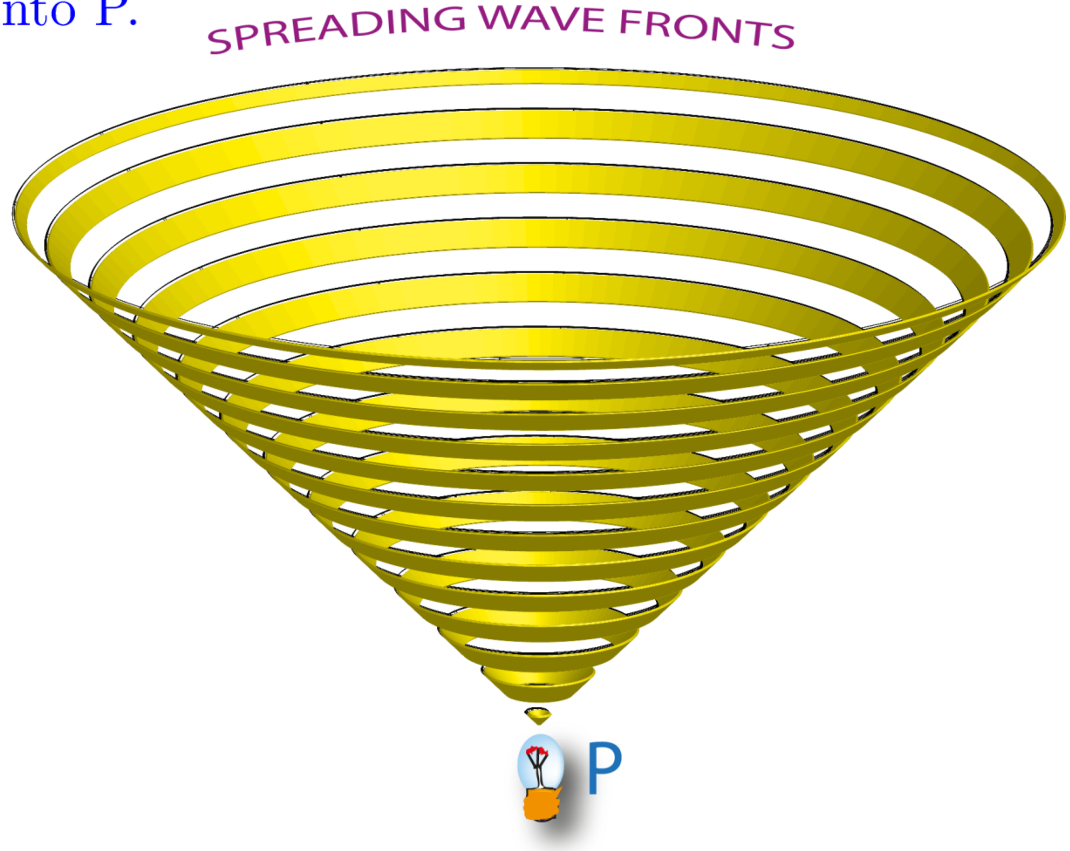
Triggering a light flash is like trowing a stone in a pond.
The spacetime history of the wavefronts of a light flash:
The Light Cone. The opening of the cone is the speed
of light in vacuum.





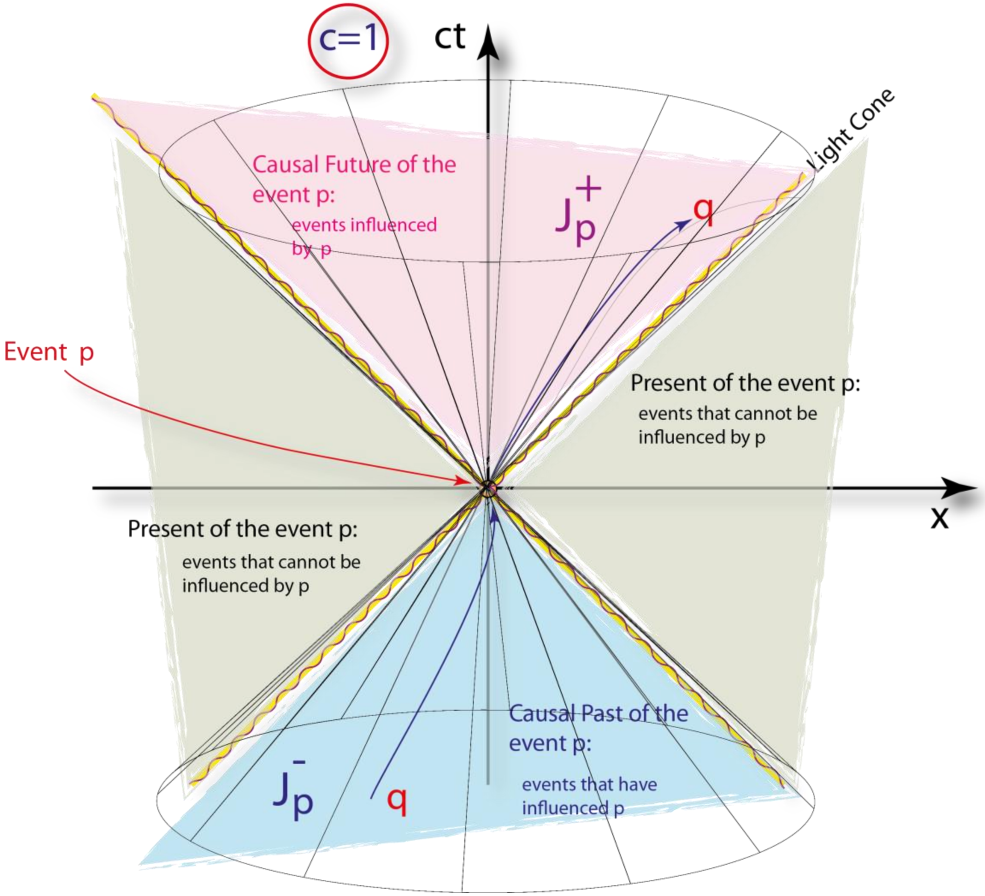
**c**eleritas = 299,792,458 metres per second

The future light cone at an event P may be thought of as the world-history of the propagation of a light-front emitted (in vacuum) by a light-flash at P. Each light-front is a 2-sphere expanding from P. The past light cone at P is the world history of a light front converging into P.



The light cone is not peculiar to electromagnetism. The propagation of every massless field is characterized by this cone. The light-cone is a *property* of Minkowski spacetime, related to its geometric properties.

The *causality principle* together with special relativity implies that no physical signal can travel faster than light. It follows that **the light cone determines the causal structure**: how spacetime events are causally related to each other.

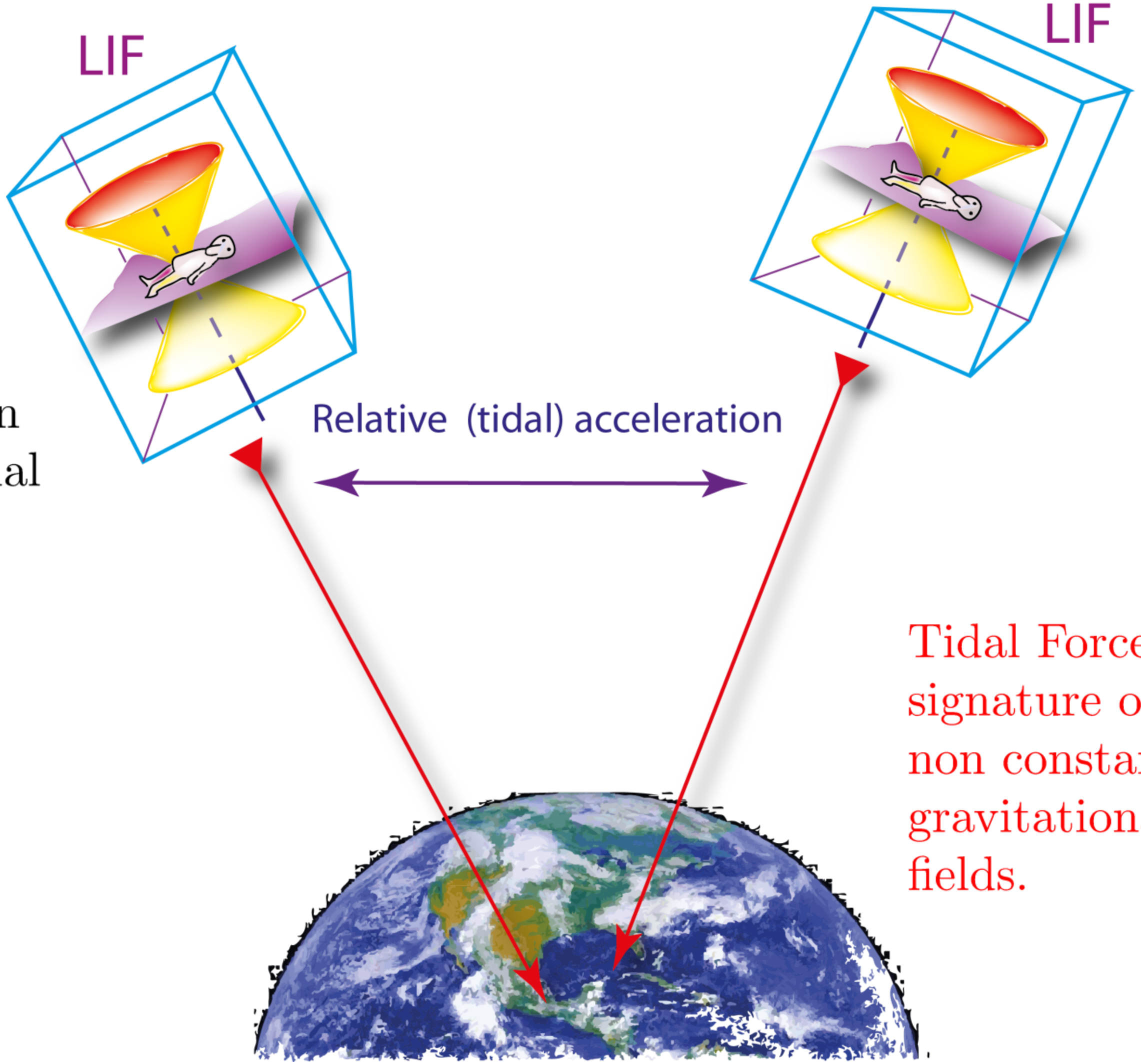


Hence, regardless of electromagnetism, the light cone can be identified as the characterization of the causal structure of Minkowski spacetime: given an event P, it tells us which event is in the *past*, in the *future*, or in the *present* of P.

A better name for the light cone would be : *The Causal Cone*

In special relativity the causal cones are all equivalent: we can map one into any other via (active) Poincare' trasformations. Hence, the causal structure of Minkowski spacetime is rigid. However, if we switch on gravity this is no longer true ...

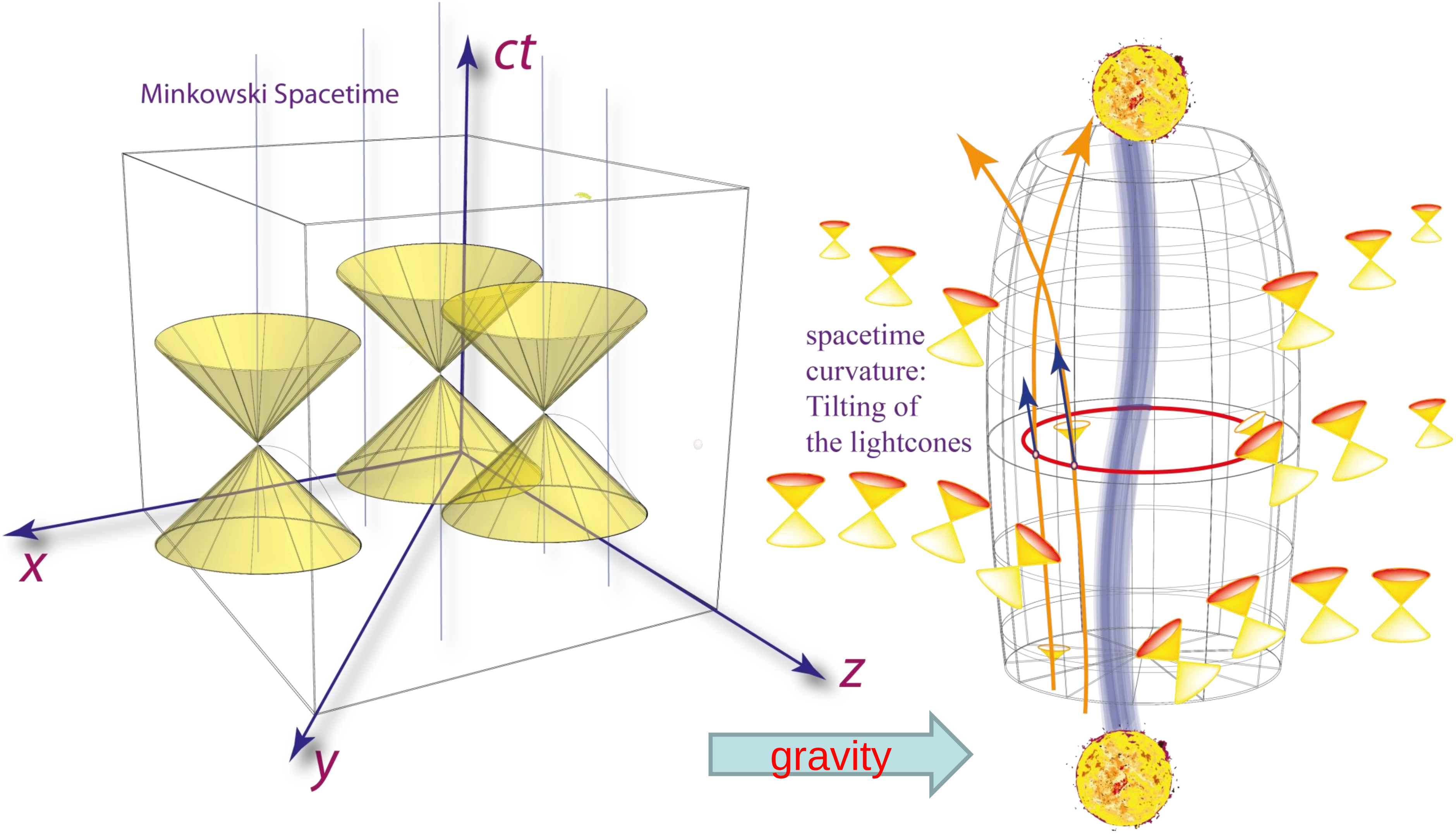
The local inertial frames can accelerate with respect to each other.



The relative acceleration between two local inertial frames is generated by the gradient of the gravitational field (Tidal forces)

Tidal Forces: signature of non constant gravitational fields.

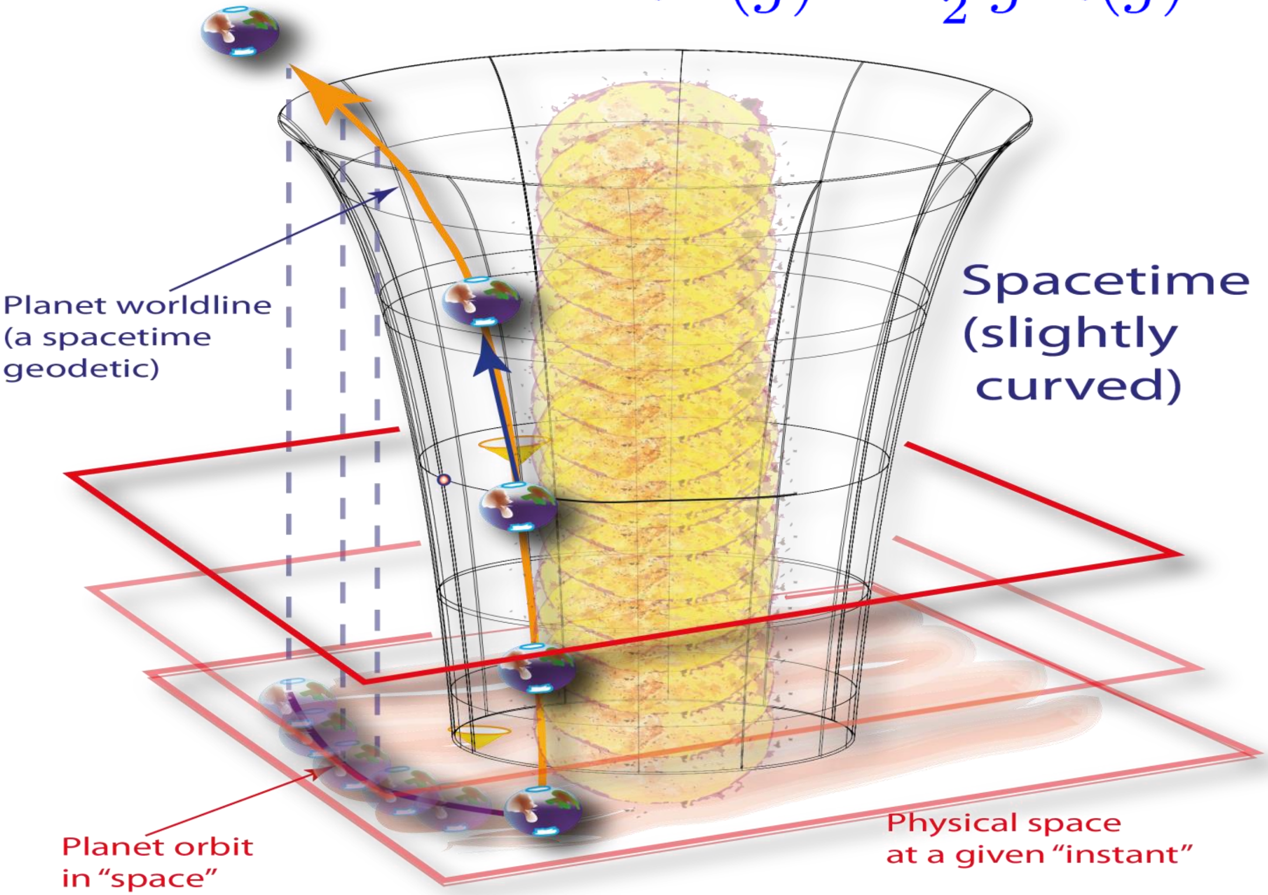
It follows that gravity deforms (and determines) the causal structure of space-time: Spacetime is causally not rigid and static: its causal properties react to all form of mass energy



spacetime
curvature:
Tilting of
the lightcones

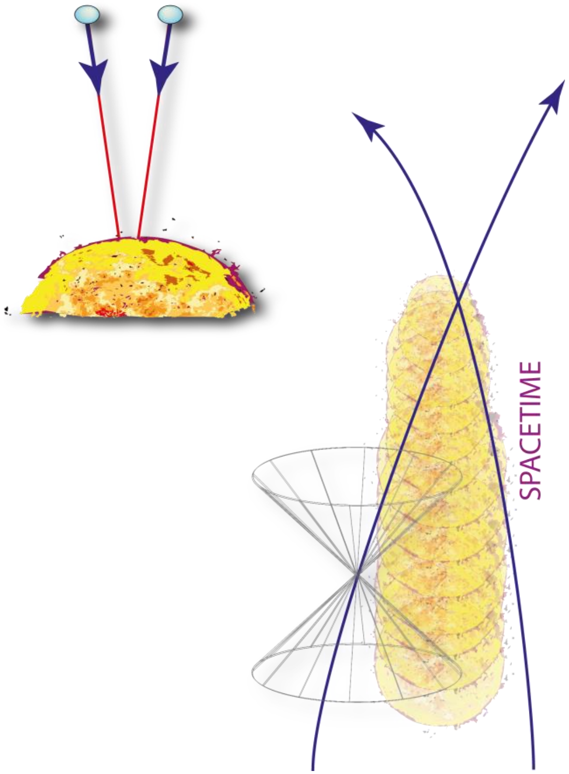
- This also implies that gravity becomes the manifestation of a dynamical spacetime geometry: This is **General Relativity**, the modern theory of gravitation governed by **Einstein's equations**:

$$Ric(g) - \frac{1}{2}\, g\, R(g) = \frac{8\pi\, G}{c^4}\, T$$



This connection between spacetime geometry and Gravity is fascinating but also leds to a severe problem: Gravitational Collapse and Singularities

(1) Gravity plays a significant role if the typical velocities $v$ induced by a mass $M$ concentrated in a region or radius $r$ are such that



$$\tfrac{1}{2}\,v^2 \;\simeq\; \tfrac{GM}{r}$$

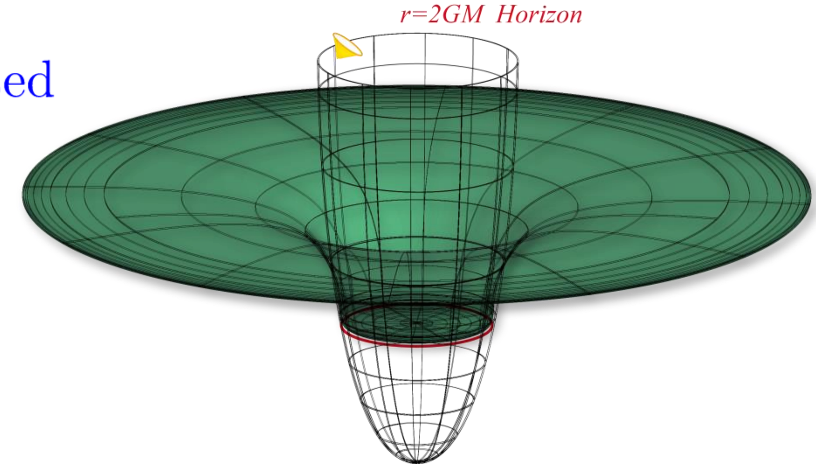
(2) Special Relativity plays a significant role if

$$v^2 \;\simeq\; c^2$$

(3) Hence General Relaivity plays a significant role if

$$\tfrac{1}{2}\,c^2 \;\simeq\; \tfrac{GM}{r}$$

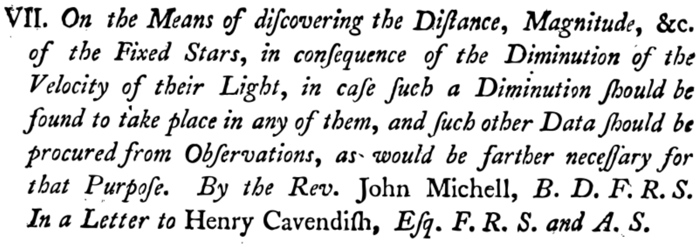
*i.e.*, if the region where $M$ is concentrated has a radius



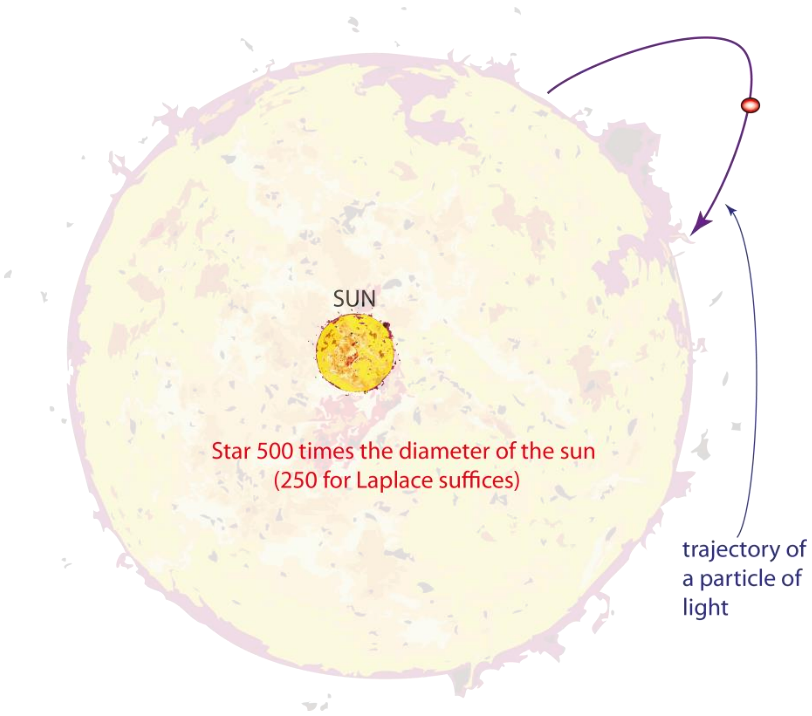
$$r \;\simeq\; \tfrac{2GM}{c^2} \;:=\; r_{Schw}$$

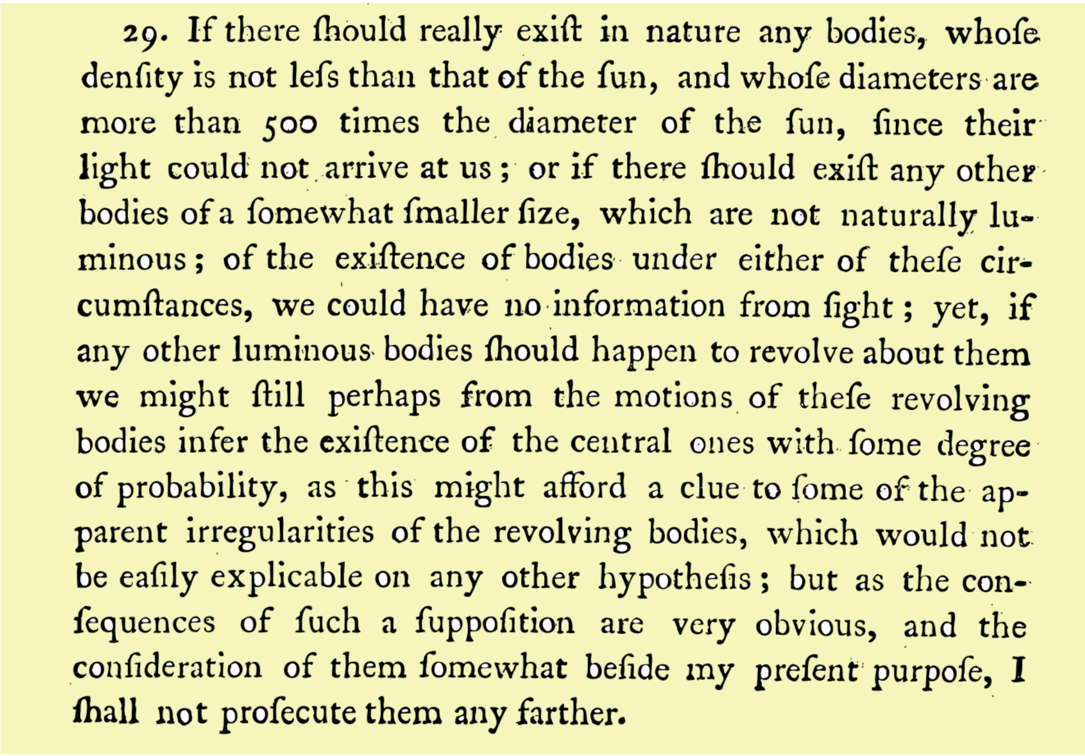
No other phenomenon put this general relativistic regime better to the fore than a collapsing star evolving into a **Black Hole**: here the connection between causal structure and gravity features at its peak.

The possibility of the existence of Black Stars (Black Holes) had been already *explored*, in Newtonian theory, by John Michell (1783), and Pierre-Simon de Laplace (1798)).



VII. *On the Means of diſcovering the Diſtance, Magnitude, &c. of the Fixed Stars, in conſequence of the Diminution of the Velocity of their Light, in caſe ſuch a Diminution ſhould be found to take place in any of them, and ſuch other Data ſhould be procured from Obſervations, as would be farther neceſſary for that Purpoſe. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendiſh, *Eſq. F. R. S. and A. S.*

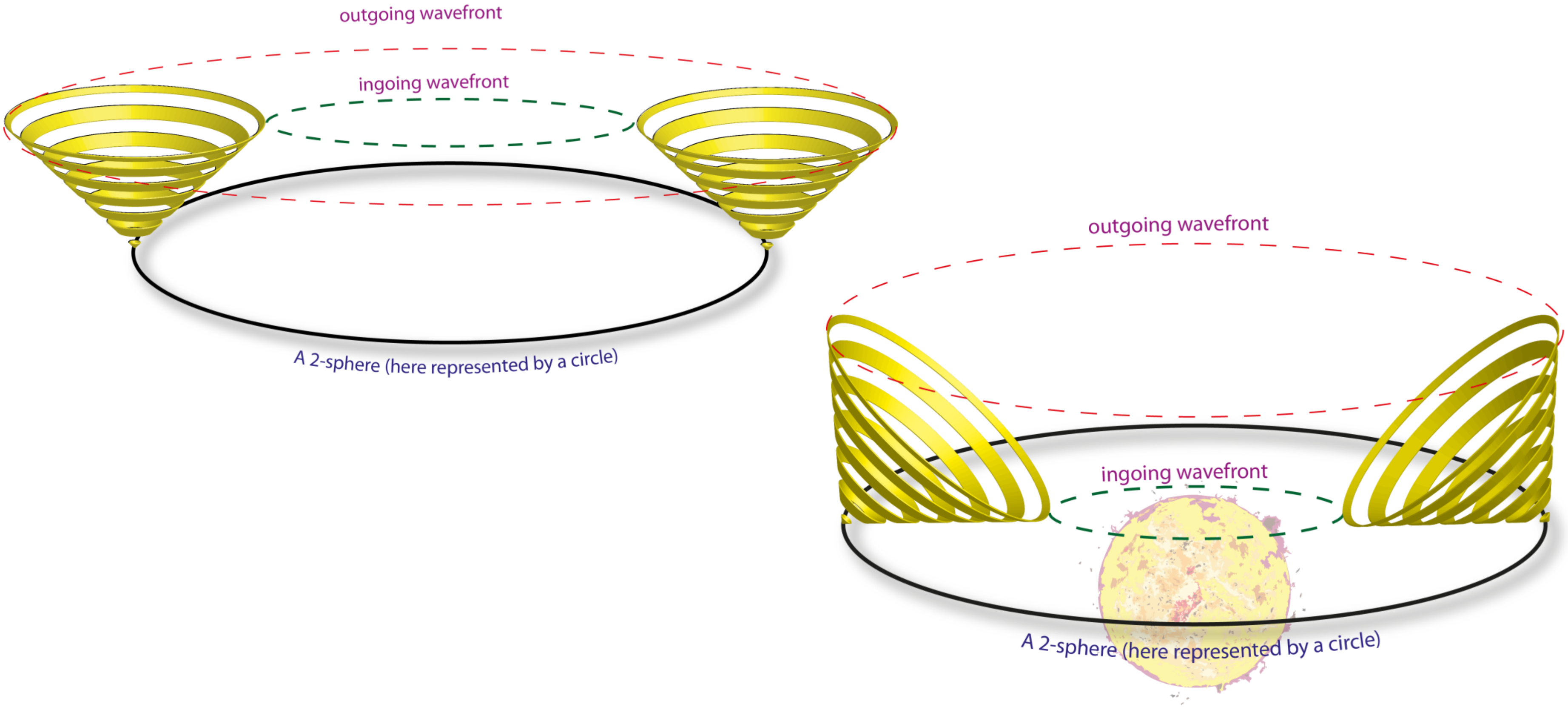




29. If there ſhould really exiſt in nature any bodies, whoſe denſity is not leſs than that of the ſun, and whoſe diameters are more than 500 times the diameter of the ſun, ſince their light could not arrive at us; or if there ſhould exiſt any other bodies of a ſomewhat ſmaller ſize, which are not naturally luminous; of the exiſtence of bodies under either of theſe circumſtances, we could have no information from ſight; yet, if any other luminous bodies ſhould happen to revolve about them we might ſtill perhaps from the motions of theſe revolving bodies infer the exiſtence of the central ones with ſome degree of probability, as this might afford a clue to ſome of the apparent irregularities of the revolving bodies, which would not be eaſily explicable on any other hypotheſis; but as the conſequences of ſuch a ſuppoſition are very obvious, and the conſideration of them ſomewhat beſide my preſent purpoſe, I ſhall not proſecute them any farther.

*Philosophical Transactions of the Royal Society of London*, Vol. 74 (1784), pp. 35-37.

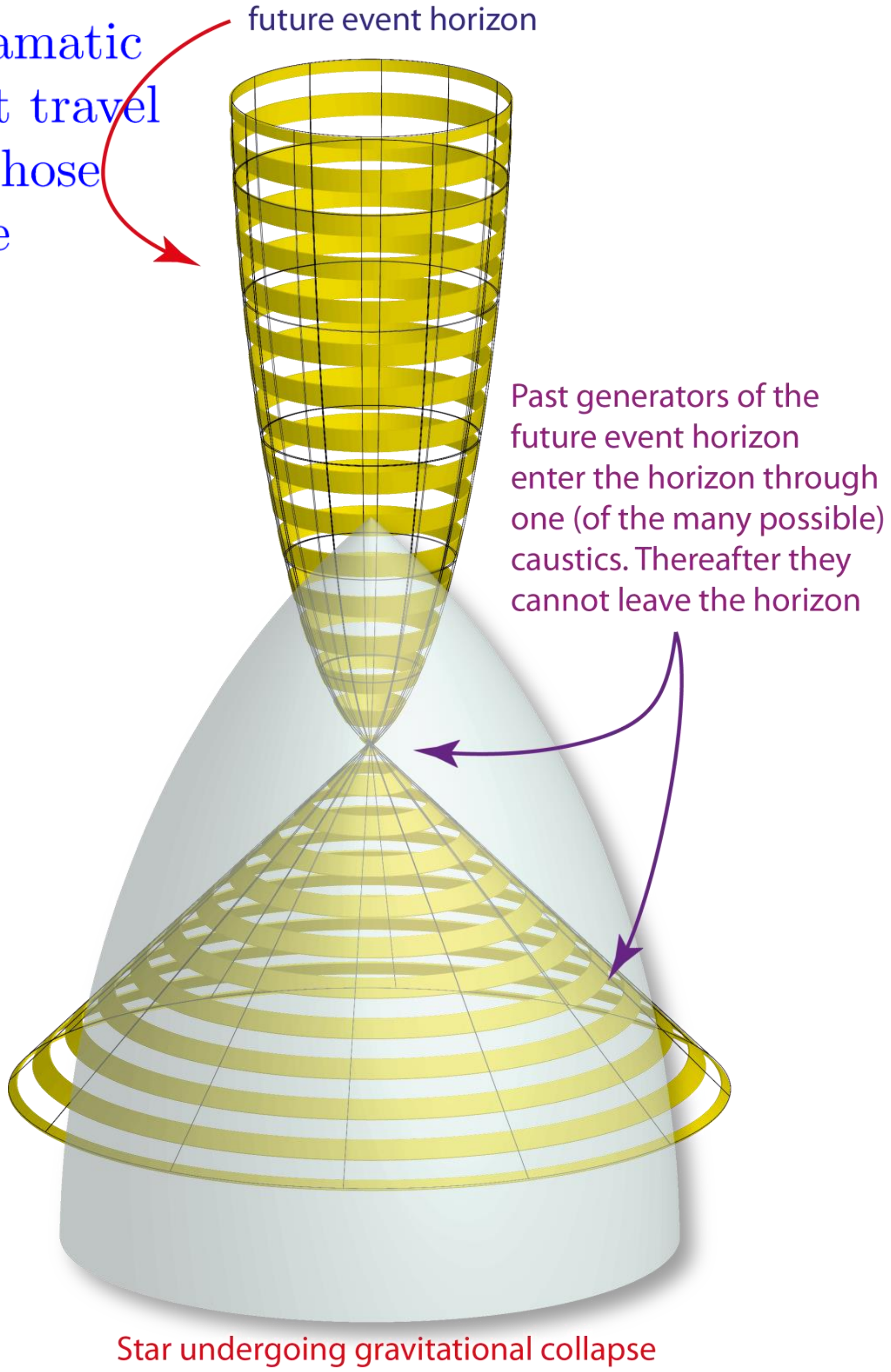
This Newtonian result, which seems to imply that ”the largest bodies in the Universe could remain invisible to us”, is not dynamically very relevant, since it focalizes on the size rather than on the density of the star.



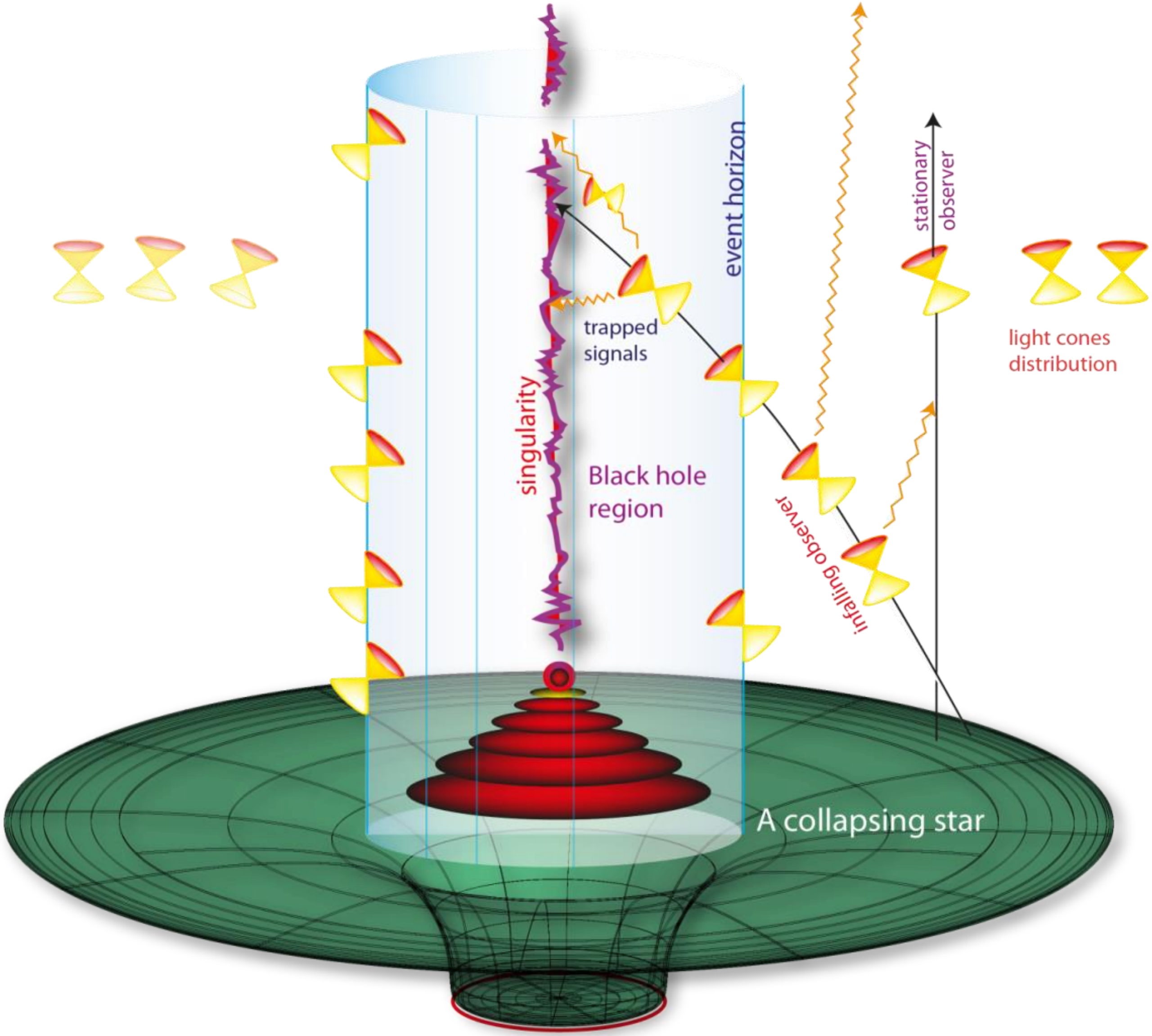
However, **according to general relativity what matters is density**, and if a sufficiently large amount of matter is concentrated in some region, it can deflect light going out from the region so much that it can be dragged back inwards

In General Relativity, this situation is dramatic since it implies that matter, which cannot travel faster than light, is trapped in a region whose boundary decreases to zero within a finite proper time. A Black Hole is born, a spacetime SINGULARITY must occur.

That this was *generically true* was proven by R. Penrose (1965). Before that, there was the work by Robert Oppenheimer and Hartland Snyder (1939) describing the gravitational collapse of a *ball of dust*. It was (erroneously) believed that the development of a singularity was an artifact of the spherical symmetry.

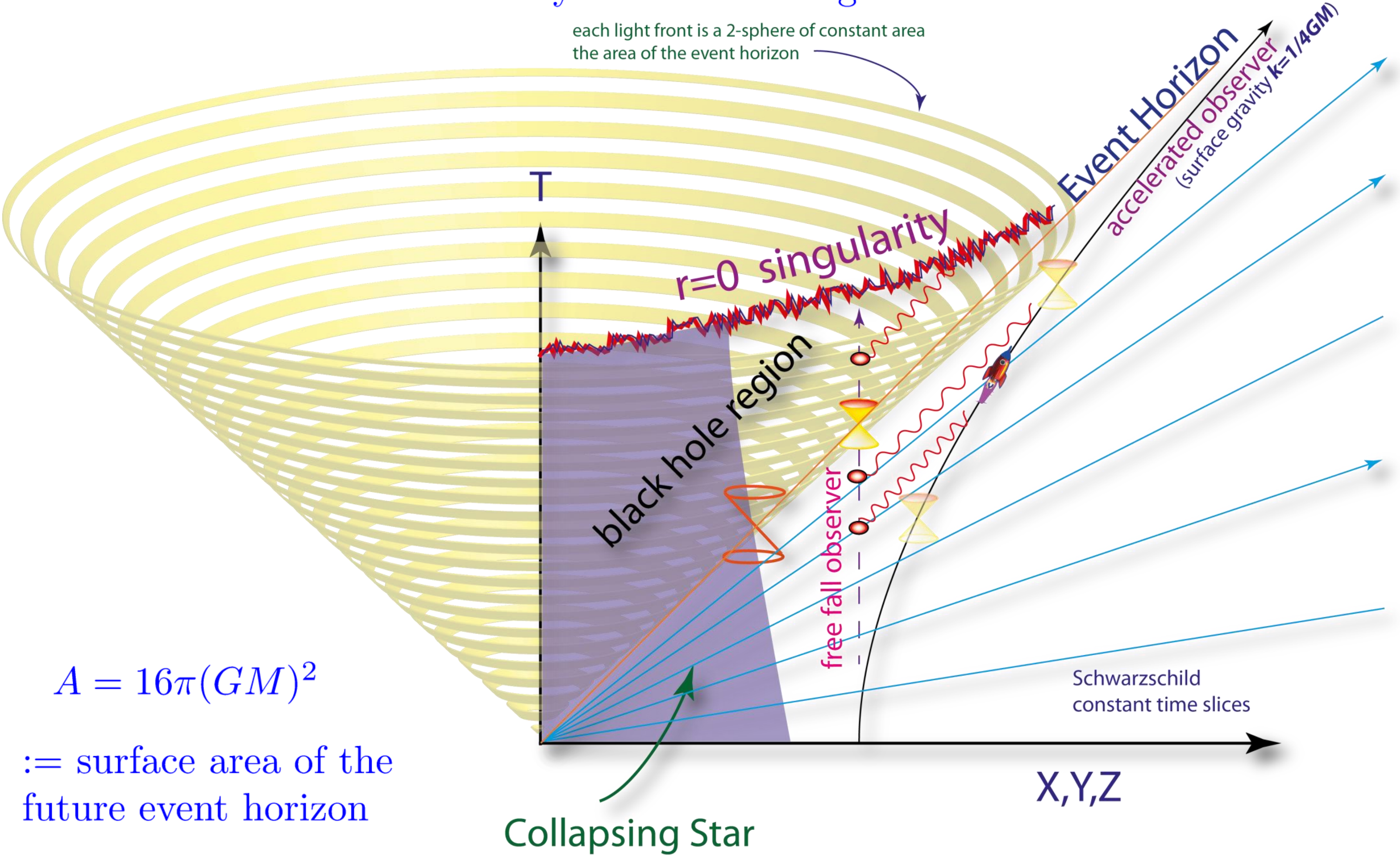


# e.g. Schwarzschild Spacetime: A non–rotating Black Hole of mass $M$



$$g = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

The **spacetime causal structure** of a Schwarzschild black hole can be more clearly seen in the diagram



$$A = 16\pi(GM)^2$$

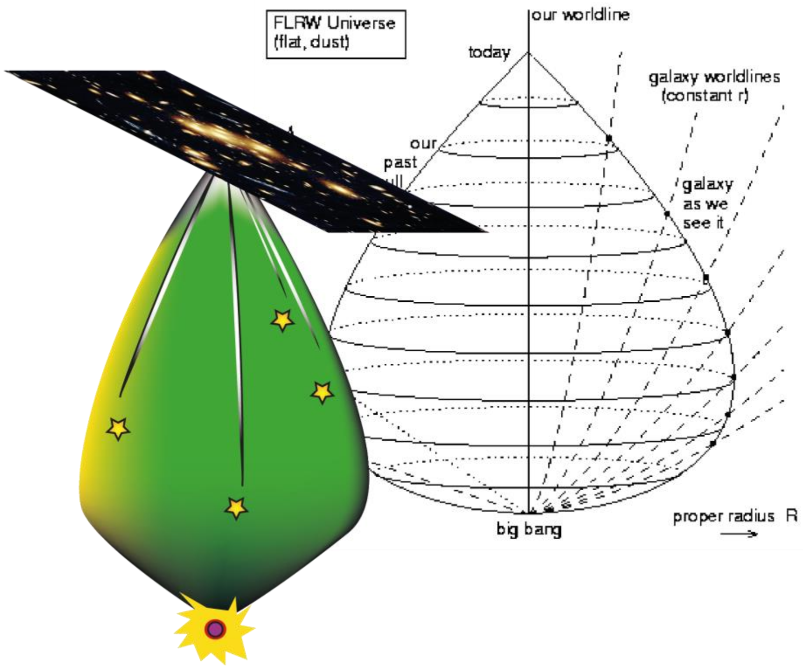
:= surface area of the future event horizon

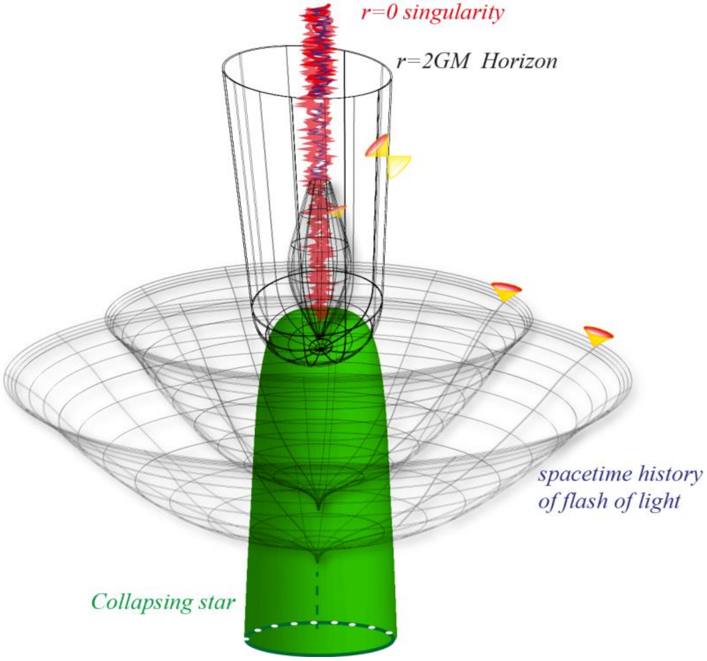
Is this the generic picture or the consequence of the particular properties of the Schwarzschild solution?

The development of singularities in spacetime implies a corresponding break down of the laws of physics, a basic issue in modern general relativity.

The two typical situations where singularity of this type occur are:

- The gravitational collapse of sufficiently massive stars.
- Cosmology: the big bang singularity.

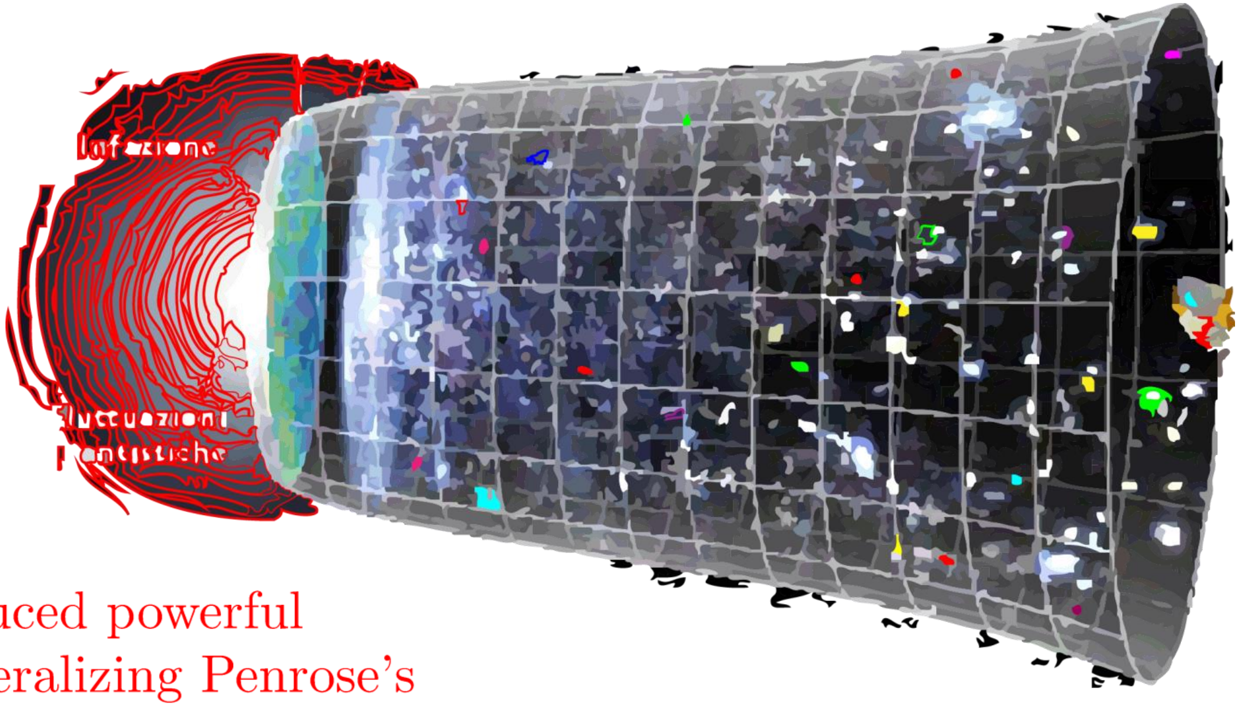




The problem of the development of singularities marks Hawking's debut in general relativity.

Hawking (with G. F.R. Ellis, (1965)): initial analysis of a cosmological singularity theorem

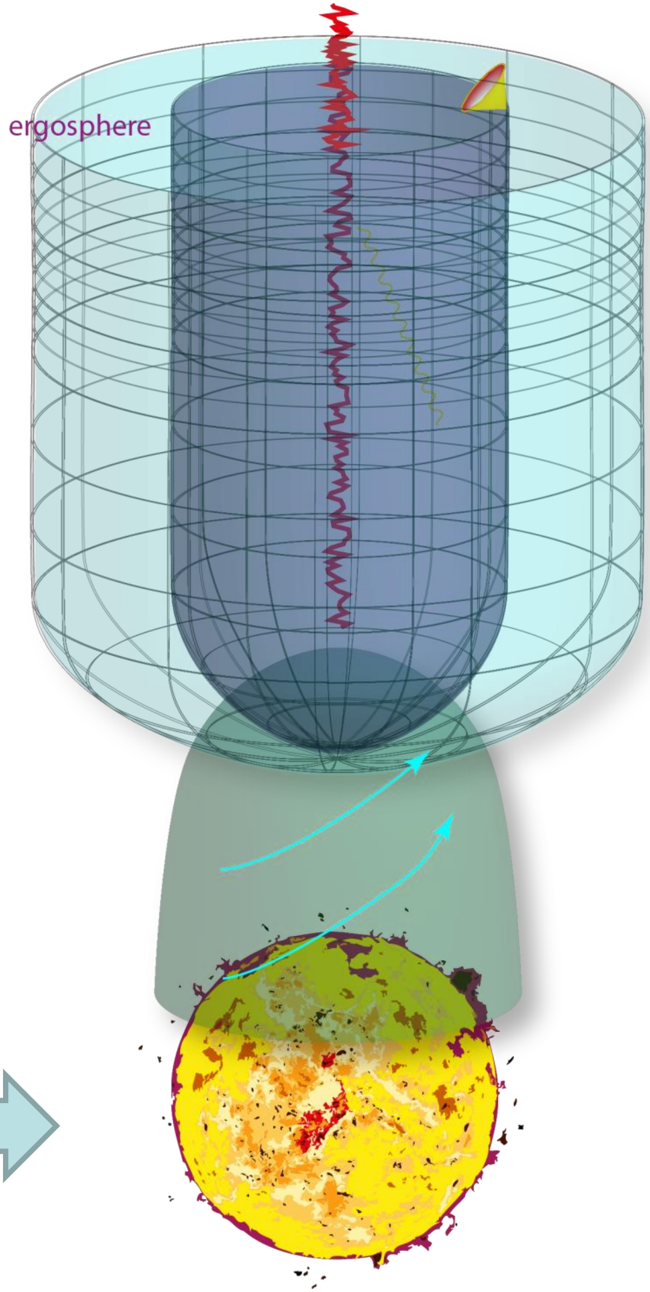
Collaboration with R. Penrose (1970) (fostered by D. Sciama). They were able to prove that a big bang space-time singularity was also a feature not just of the standard highly symmetrical Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological models, but also of more generic (not symmetric) model



Eventually, he produced powerful theorems vastly generalizing Penrose's results by showing that a big-bang-type singularity was a necessary implication of general relativity.

He contributed (1972) in a fundamental way to a basic result of Black Hole theory:

- Non-rotating black holes, when they had finally settled down to become stationary, would necessarily become completely spherically symmetrical, the corresponding spacetime geometry being that described by the Schwarzschild solution (W. Israel, 1967).
- Similarly for Rotating black holes, which settle down to the stationary axial symmetric Kerr spacetime metric (B. Carter, D. Robinson): these are the black holes that we find in nature.



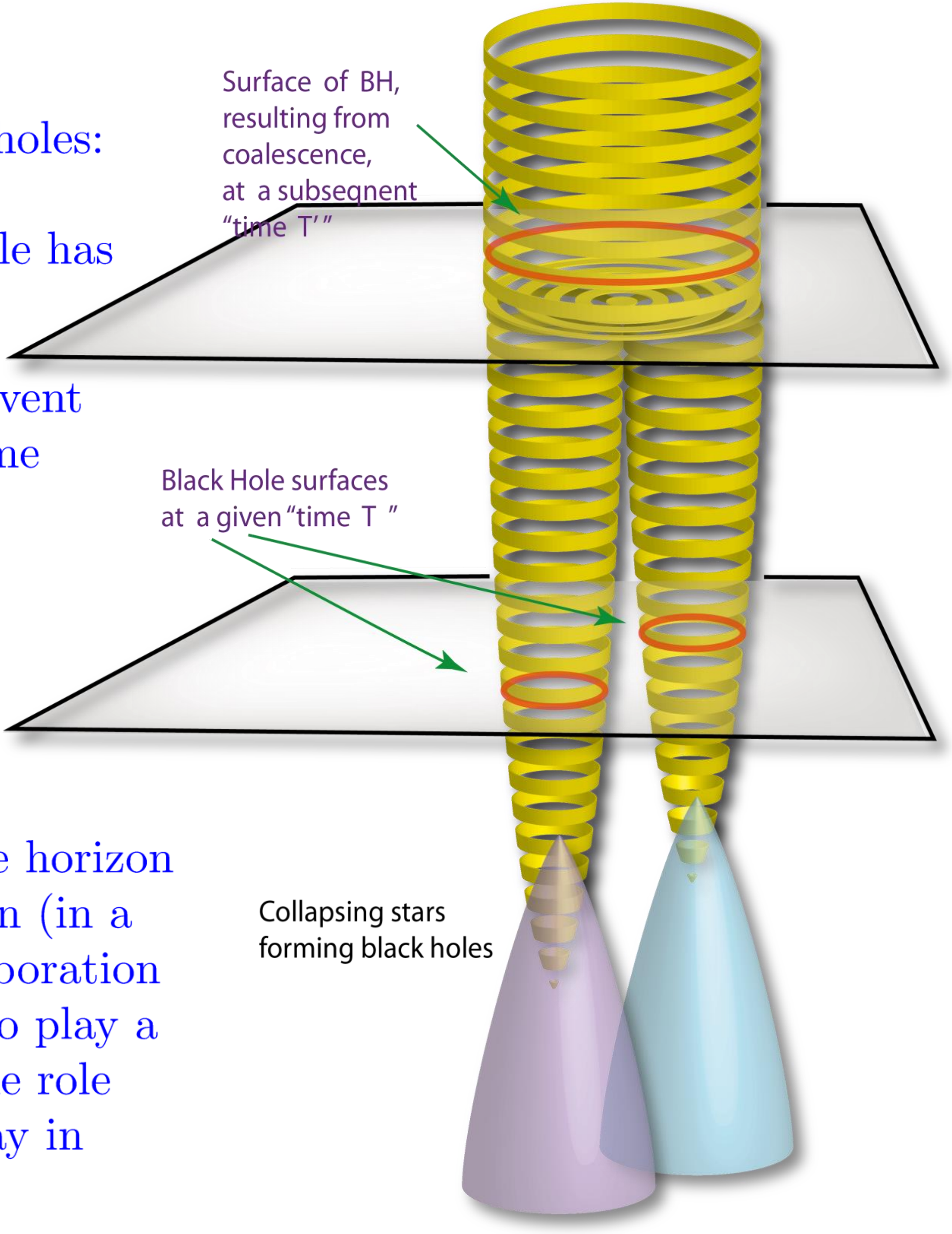
A rotating black hole

Hawking established a number of fundamental results about black holes:

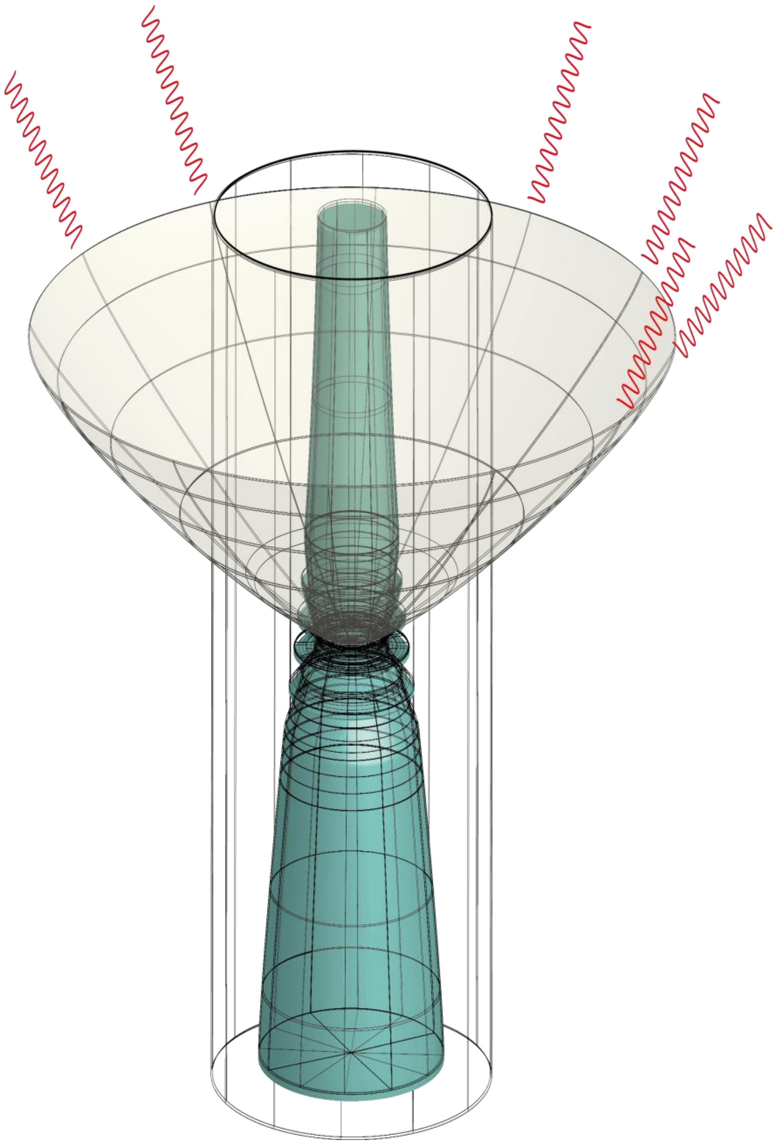
- The event horizon of a black hole has the topology of a sphere
- The surface area of the future event horizon never decreases with time (under rather weak conditions).

These properties suggested him a deep analogy between the behaviour of black holes and the laws of thermodynamics:

The surface area of the black hole horizon and its surface gravity were shown (in a fundational work written in collaboration with B. Carter and J. Bardeen) to play a role analogous, respectively, to the role that entropy and temperature play in thermodynamics



# The analogy was suggested by *The laws of Black Hole mechanics*



$$A = 16\pi(GM)^2$$

**0th Law**: the *surface gravity* of the (future) event horizon $\kappa$ is constant

$$\kappa := \frac{\sqrt{G^2M^2 - a^2}}{2GM\left(GM + \sqrt{G^2M^2 - a^2}\right)}$$

**1st Law**: if a stationary $BH(M, J)$ is perturbed to another stationary $BH(M + \delta M, J + \delta J)$ then

$$dM = \tfrac{\kappa}{8\pi} dA + \Omega_H \, dJ$$

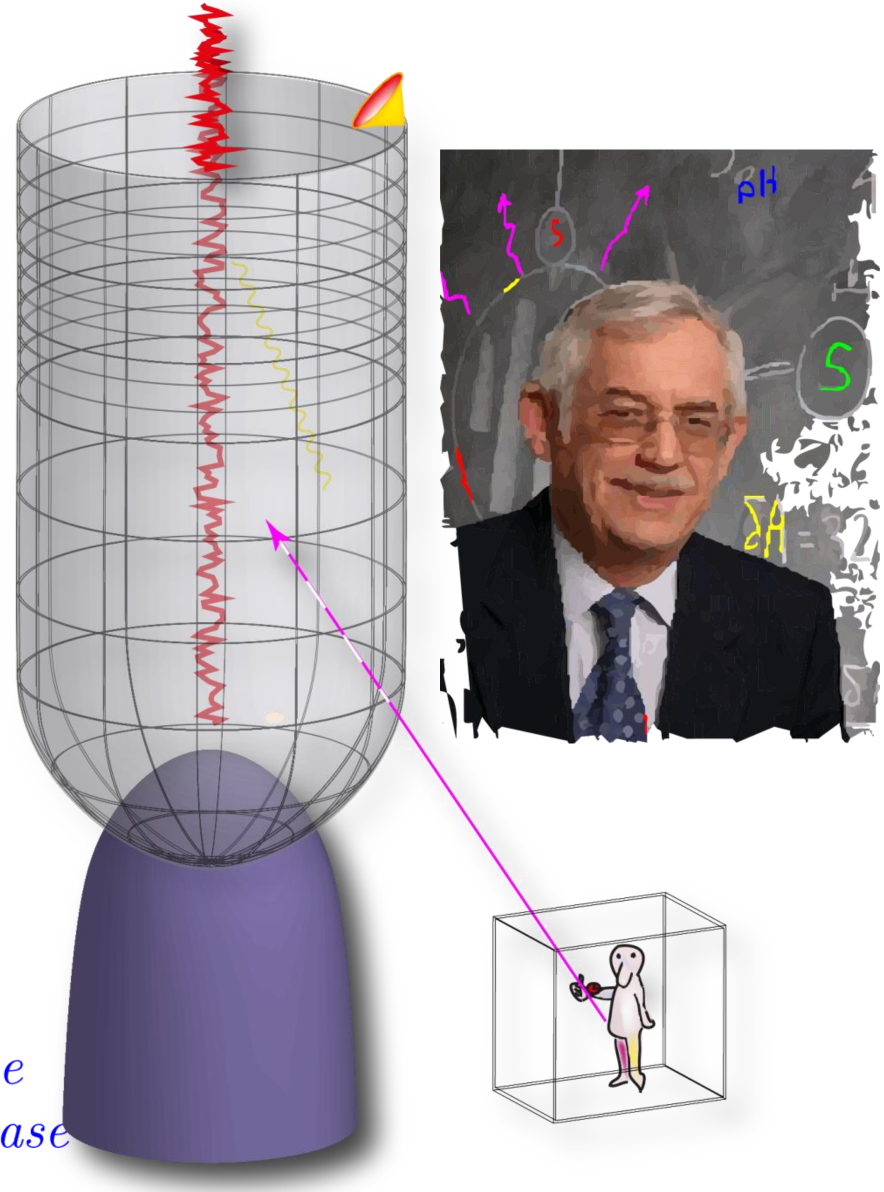
$\Omega_H =$ BH *horizon angular velocity*

$$= \frac{J}{2GM\left[GM^2 + \sqrt{G^2M^4 - J^2}\right]}$$

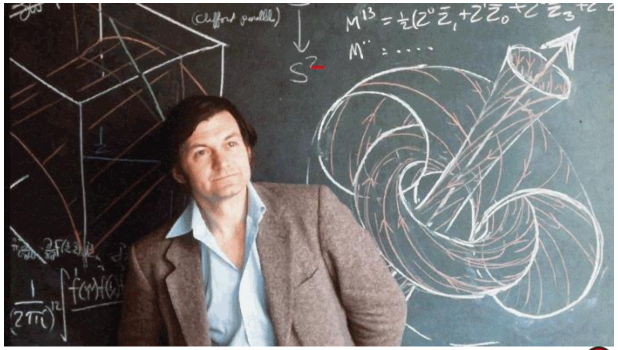
**2nd Law**: Hawking's area theorem holds: *The surface area of the future event horizon never decreases with time*

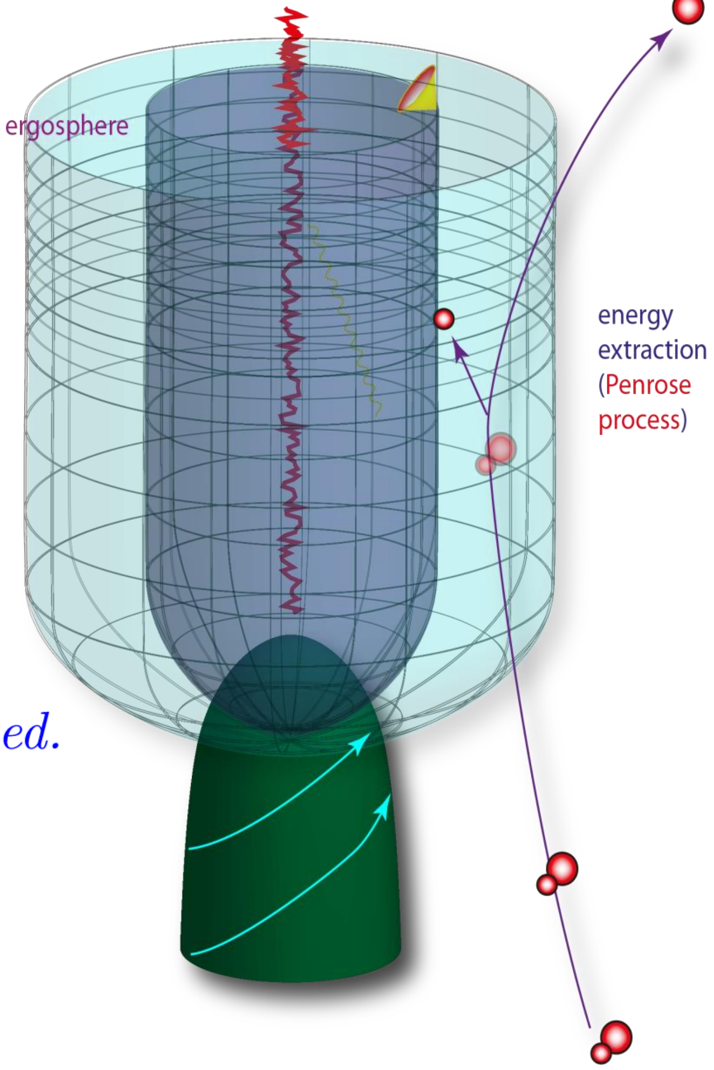
| Law | Thermodynamics | Black Holes |
|---|---|---|
| 0th | $T$ constant for system in thermodynamical equilibrium | $\kappa$ Constant on stationary BH horizon |
| 1st | $dE = TdS + pdV$ | $dM = \frac{\kappa}{8\pi} dA + \Omega_H \, dJ$ |
| 2nd | $\delta S \geq 0$ in every process | $\delta A \geq 0$ in every process |
| 3rd | Impossible to get $T=0$ in a physical process | Impossible to get $\kappa = 0$ in a physical process |

This thermodynamic behaviour of black holes seemed, at the time, just a curiosity...*But it was Jacob Bekenstein, who took the bold step, of suggesting the area actually was the physical entropy, and that it counted the internal states of the black hole. I was very much against this idea at first, because I felt it was a misuse of my horizon area result. If a black hole had a physical entropy, it would also have a physical temperature. If a black hole was in contact with termal radiation, it would absorb some of the radiation, but it would not give off any radiation, since by definition, a black hole was a region from which nothing would escape. If termal radiation was at a lower temperature than the black hole, the loss of entropy down the black hole, would be greater than the increase of horizon area.* (Hawking)



*… No one, including Bekenstein and myself, thought anything could get out of a non rotating black hole. On the other hand, Penrose had shown that energy could be extracted from a rotating black hole, by a classical process. This indicated that there should be a spontaneous emission in the super radiant modes, that would be the quantum counterpart of the Penrose process. In trying to understand this emission in the super radiant modes, in terms of quantum field theory in curved spacetimes, I stumbled across the fact that even non rotating black holes, would radiate. Moreover, the radiation would be exactly what was required, to prevent a violation of the second law. Bekenstein was right after all, but in a way he hadn't anticipated.* (Hawking)





Black hole physics connects the Area Theorem to the second law of Thermodynamics: $dS \geq 0$

$$S_{BH} = \frac{k_B A}{4G\hbar} = \frac{k_B \pi R_0^2}{G\hbar} = \frac{k_B\, A}{4\, l_P^2}$$

Coupling quantum matter fields to a Black Hole: emission of black body radiation at a temperature (Hawking)

$$T_{BH} = \frac{\hbar}{8\pi G k_B M}$$

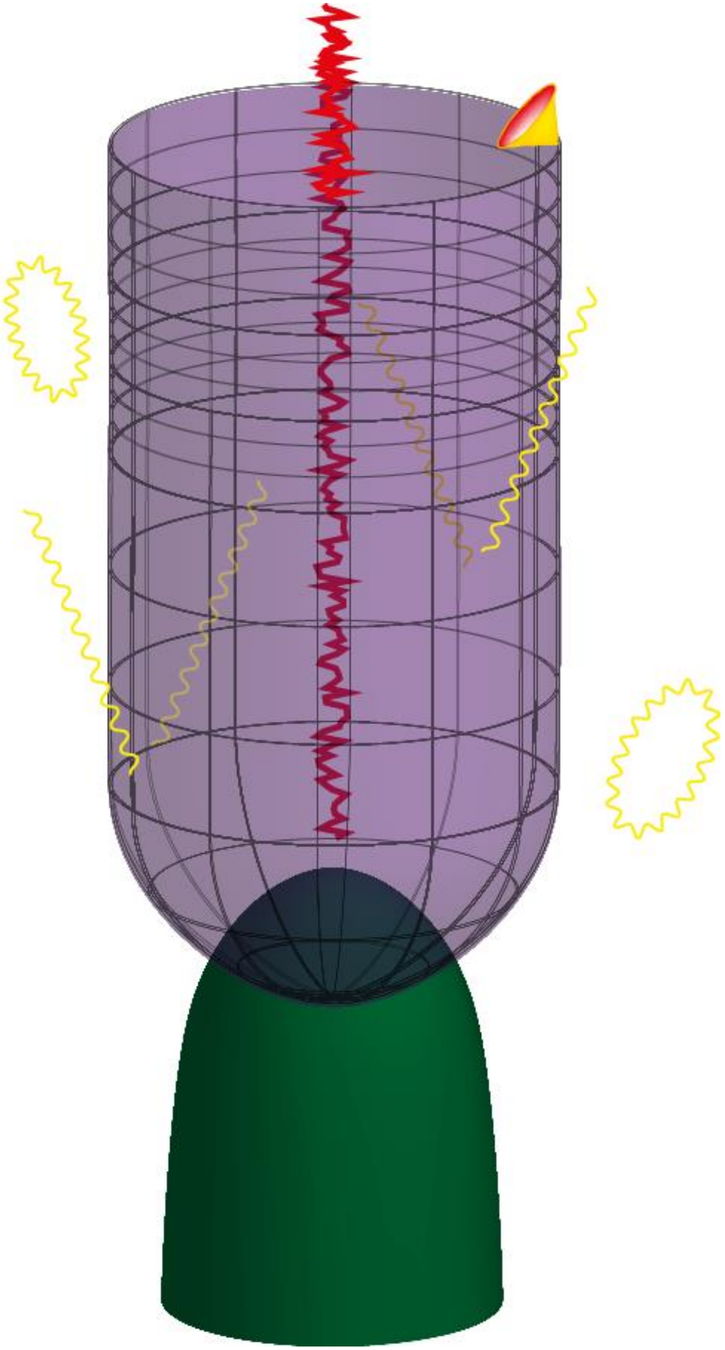
Bekenstein–Hawking (black hole) entropy

$T_{BH}(\text{few Sol M}) \simeq 10^{-8} K$

$T_{BH}(\text{Moon}) \simeq 1\, K$

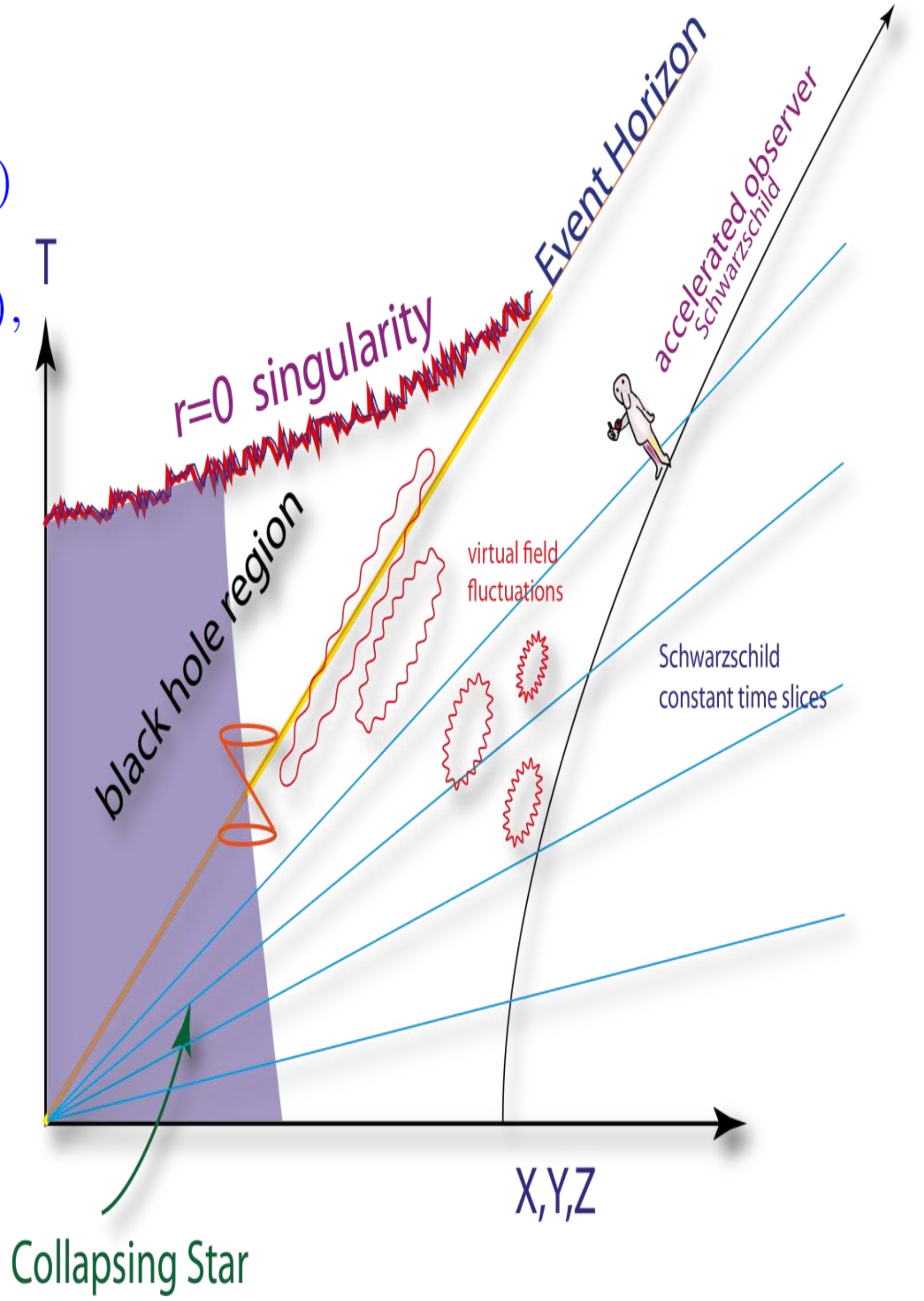
$T_{BH}(\text{Boulder}) \simeq 10^{18}\, K$

$T_{BH}(M_{Planck}) \simeq 10^{32}\, K$



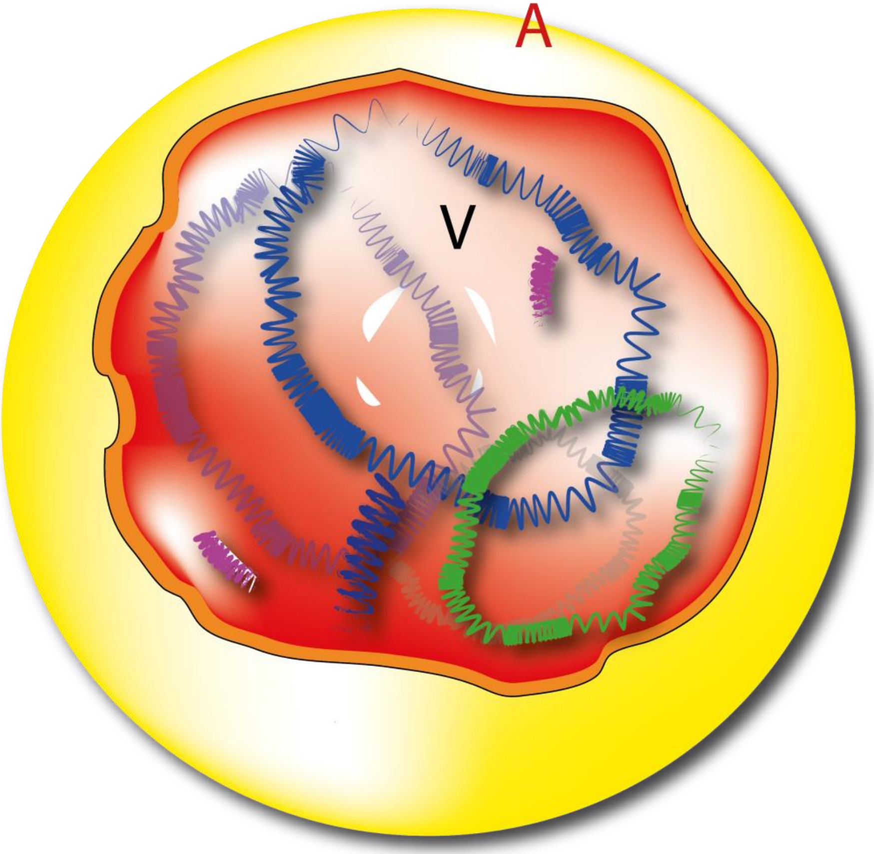
# Horizon heuristics ...

- If the horizon is interacting with a physical field: (*e.g.* a quantum field) as the field approaches the horizon its fluctuations (even the virtual ones), as seen by a Schwarzschild observer, slow down more and more as the field approaches the horizon
- Short–lived virtual fluctuations are stretched over the full surface horizon area and become physical fluctuations (a virtual particle that exists for an arbitrarily long time ...is an ordinary particle..)



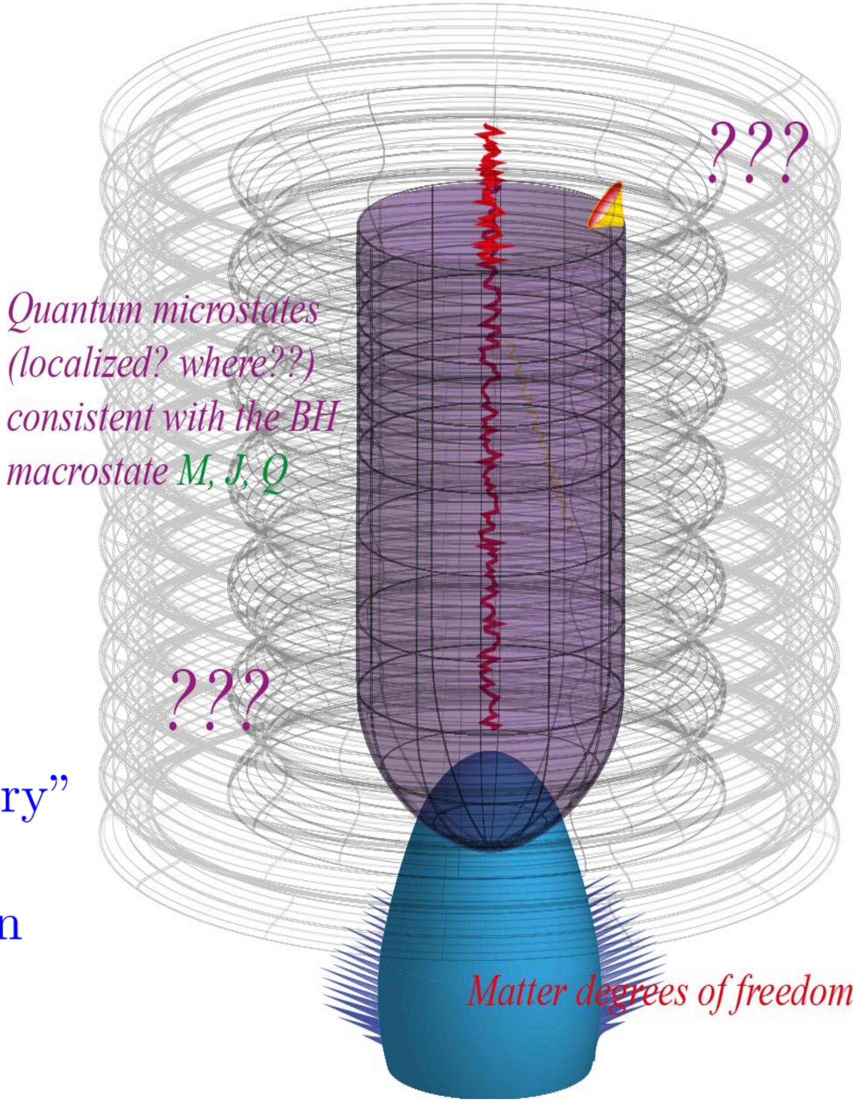
Note the basic fact that $S_{BH} \propto A$ **and not to the 3-dimensional volume** $V$ enclosed within the (horizon) surface area $A$. This is a basic feature of the thermodynamics of a system involving gravity. For a usual thermodynamical system associated to a local QFT, we have $S \propto V$. In particular, the number of QFT states with a maximum energy density $\epsilon_{max}$ that can be gathered in a $3d$ volume $V$ typically is $N_{QFT} \propto \exp[s(\epsilon_{max})\, V]$, where $s(\epsilon_{max})$ is the entropy density as a function of the energy(density)

**$S_{BH}$ is the Maximum Entropy**
of a region $V$ of 3D space: An Entropy that is proportional to the surface area measured in Planck units



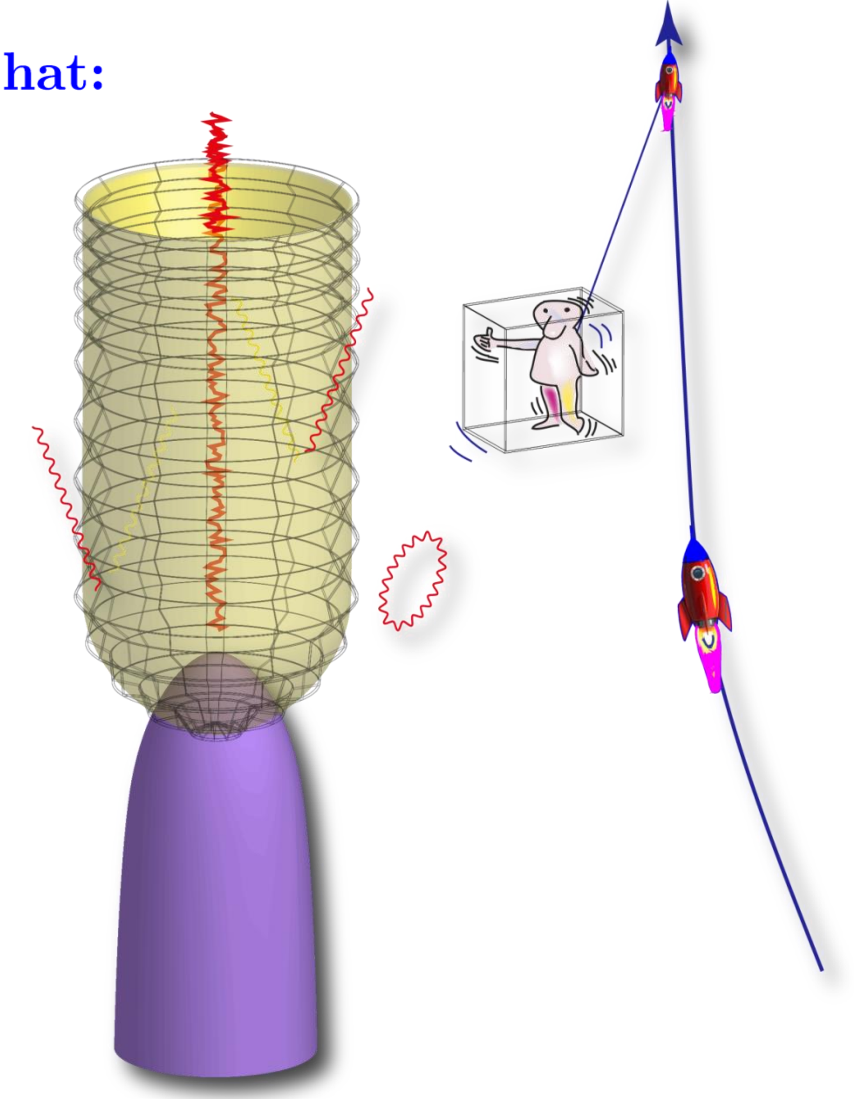
$S_{BH}$ is much larger than the physical (mass–energy) entropy of the collapsing star generating the black hole. What is its origin? Is it possible to derive $S_{BH}$ from (quantum) statistical mechanics?

To provide answers to these issue Hawking attacked the problem of quantum gravity. A formidable, not yet solved task that requires the quantisation of the very structure of spacetime. (Hawking radiation ”simply” requires a mastery of quantum fields on a classical curved spacetime



He developed a very deep approach (together with J. Hartle): the ”No-Boundary” stategy which contemplates a dynamical transition from Riemannian to a Lorentzian geometry so as to avoid the development of (cosmological) singularities.

# One may think that:



- The Mathematical entropy of a BH is a measure of the potential capacity of the BH to store information
- The physical entropy is the log of the number of microstates of the degrees of freedom that make up the physical BH
- This number is of the order of the surface area of the event horizon as expressed in Planck units
- Information can be stored, thermalized, and emitted back as Hawking radiation.

**STILL ...** ...we do not have a complete theory and Hawking radiation is challenging us, by offering a great opportunity to probe Gravity and the Quantum.

Stephen Hawking was arguably one of the greatest relativist of the last fifty years. We just touched upon his many impressive, if not revolutionary, contributions to our understanding of *General Relativity* and of their impact on the physics of the Universe.

He had a public, romanticised, image, *the symbol of the triumph of mind over matter* (R. Penrose). An image fully justified by the fact that, notwithstanding his physical condition, he was able to establish extraordinary representations of the patterns of the world, which thrive on surprise and beauty.