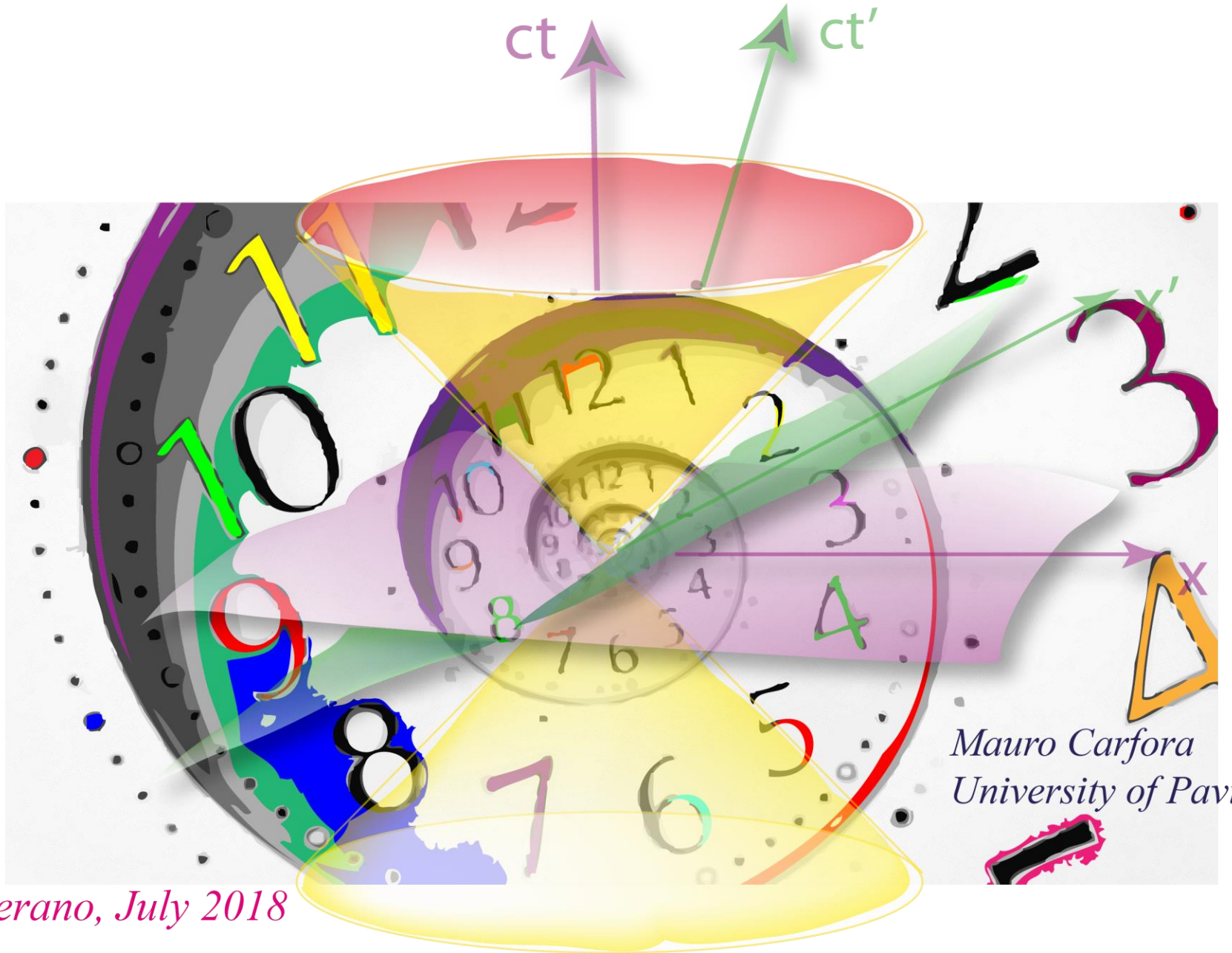
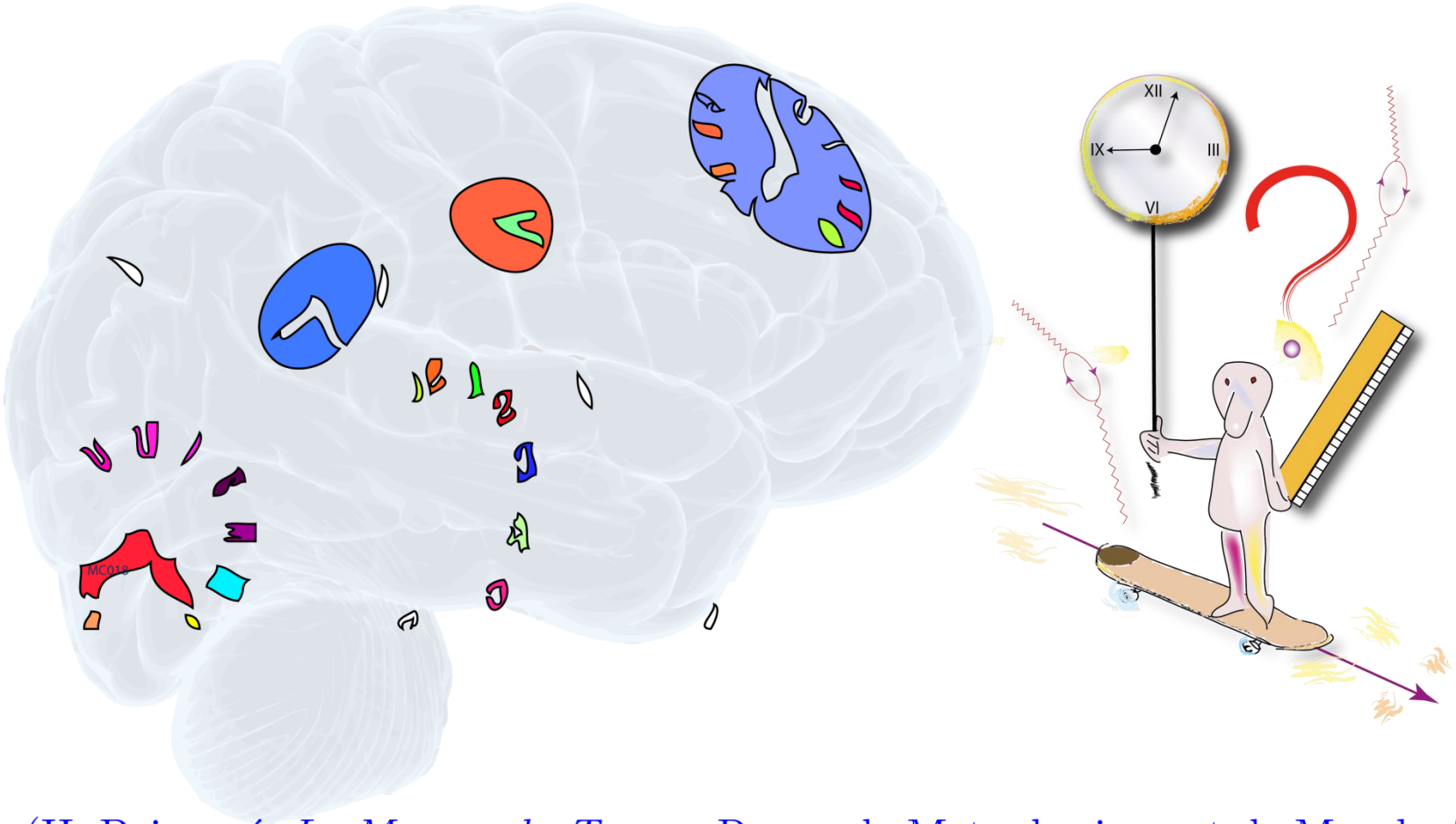
The Measurements of Time



Merano, July 2018

- The paradigm of Time we experience :

Tant que l'on ne sort pas du domaine de la conscience, la notion du temps est relativement claire ... ... le temps psycologique nous est donné et nous volulons créer le temps scientifique et physique. C' est la que la difficulté commence, o plutot les difficultés, ...

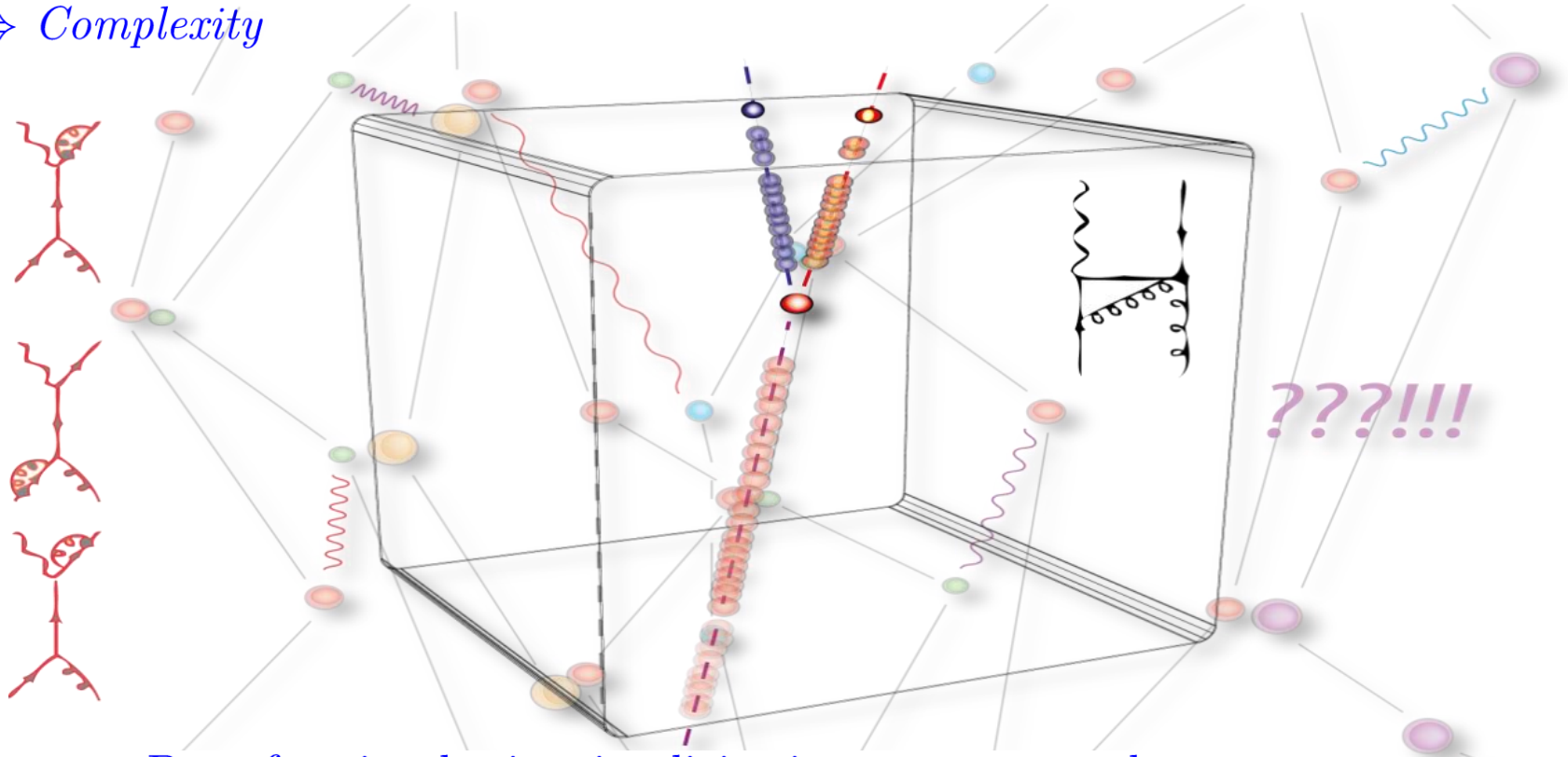


(H. Poincaré, *La Mesure du Temps*, Revue de Metaphysique et de Morale, T. 6, No. 1 (Janvier 1898), pp. 1-13, Presses Universitaires de France)

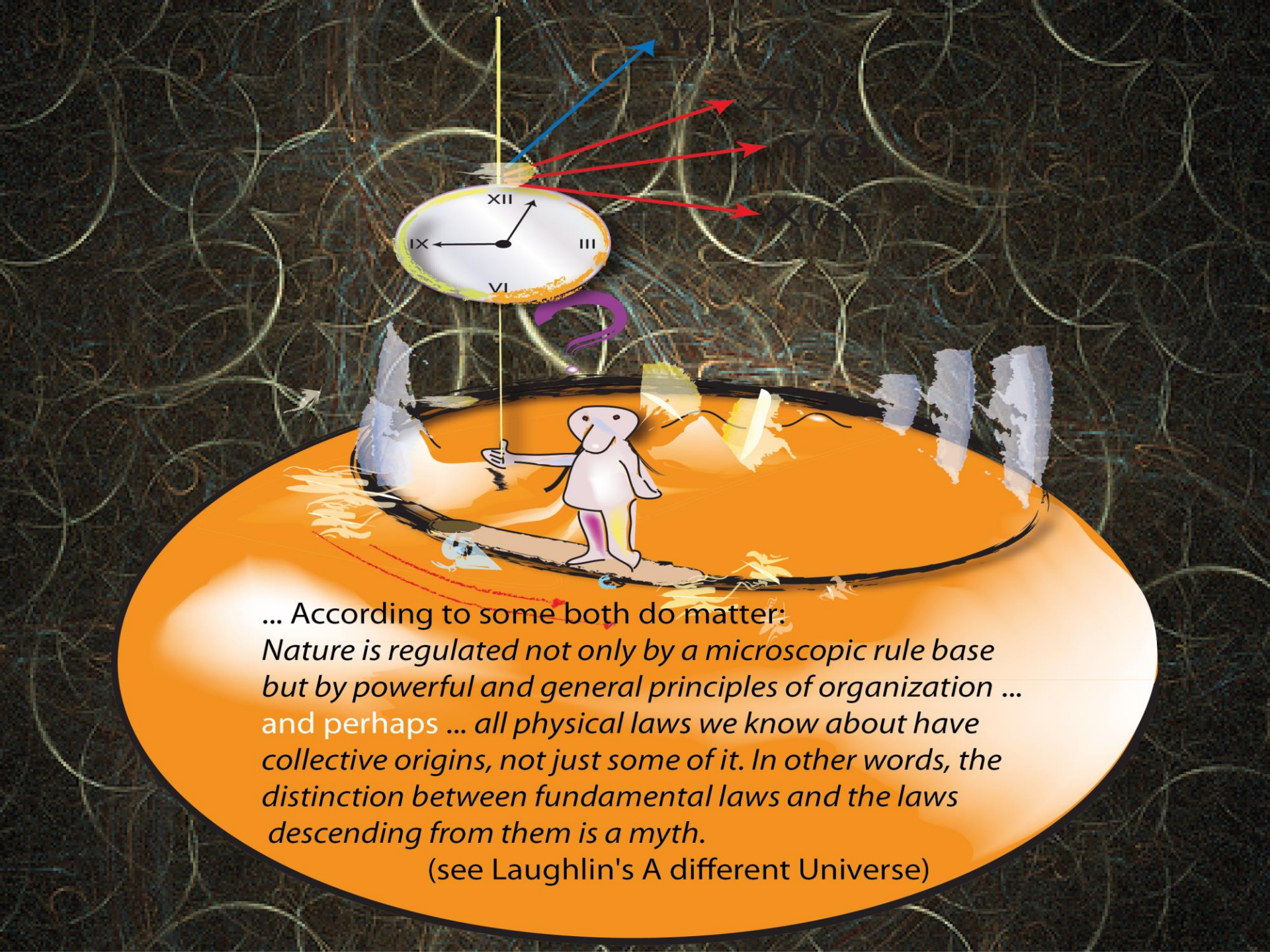
What is physical time? What is its central role in the behavior of physical systems? Where does the *Arrow of Time* come from?

The *reductionist* point of view: Discuss Time in terms of the fundamental laws of Classical (Newtonian and Relativistic) Physics and in relation to Quantum Theory. $\Longrightarrow$ *Simplicity*

The *emergent* point of view: Discuss (the arrow of) Time as an emergent collective phenomenon consequence of the interactions among the costituent parts. $\Longrightarrow$ *Complexity*



But often in physics simplicity is an emergent phenomenon

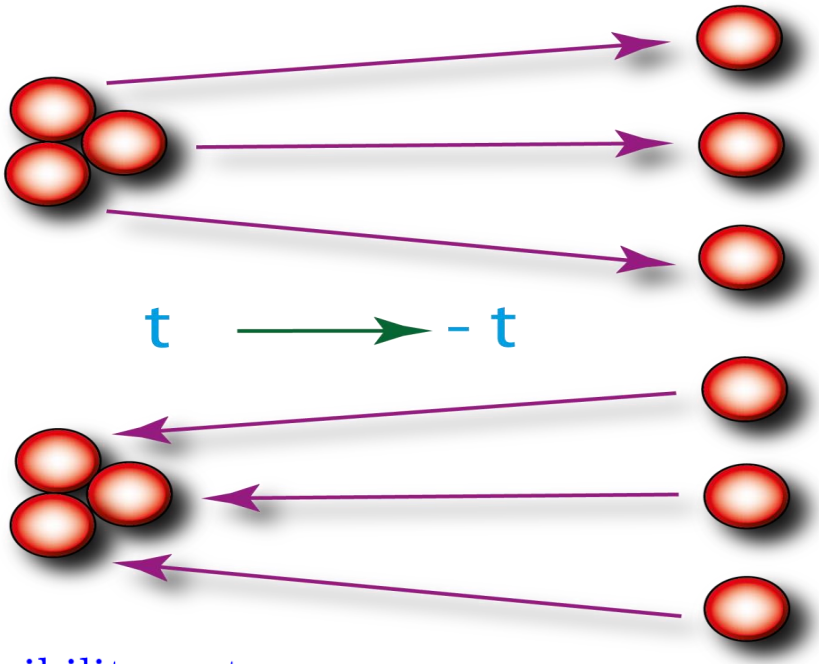


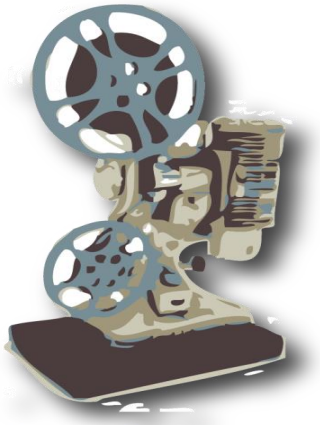
... According to some both do matter:
Nature is regulated not only by a microscopic rule base but by powerful and general principles of organization ...
and perhaps ... *all physical laws we know about have collective origins, not just some of it. In other words, the distinction between fundamental laws and the laws descending from them is a myth.*
(see Laughlin's A different Universe)

The reductionist point of view emphasizes the nature of the fundamental physical laws (classical, relativistic, and quantum). These laws are typically deterministic and time-reversible (*e.g.* Newton's equation of motion $m\,\vec{a} = \vec{F}$) as long as we do not consider dissipative phenomena (*e.g.* friction): under the exchange $t \mapsto -t$ the physical system has an evolution in the past which is deterministic like the evolution towards the future.

Link 1

Film the evolution





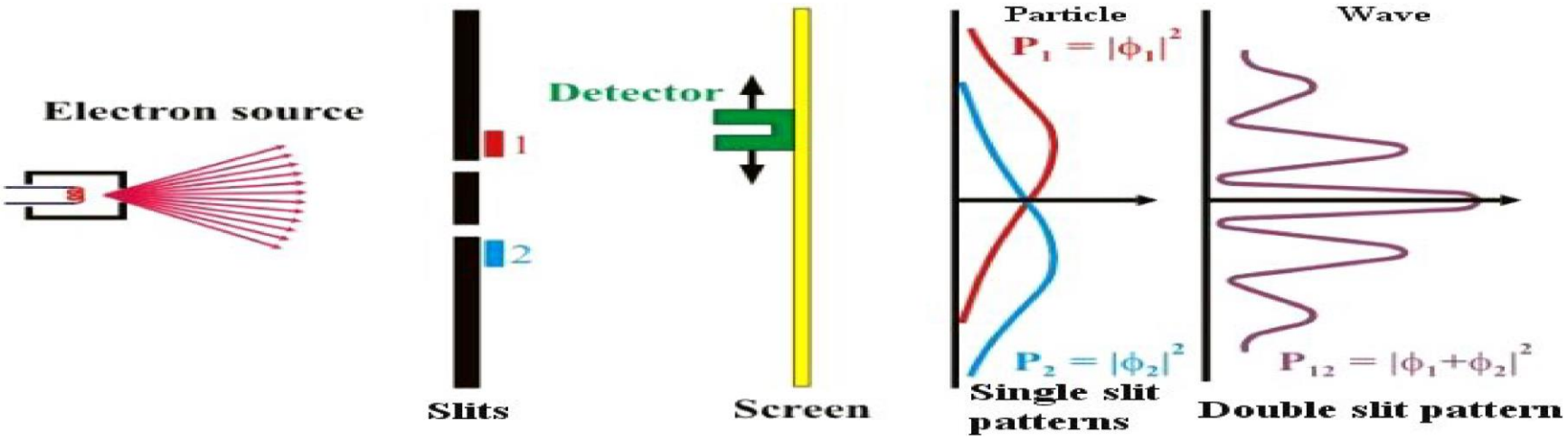
play it backwards

Hence, time-reversibility puts on the same dynamical level *prediction* and *retrodiction*.

Is often argued that time–reversal invariance fails with Quantum Mechanics. This is not true as long as we consider the free quantum flow described by the Schrödinger equation

$$i\,\hbar\,\frac{\partial\,\psi}{\partial t}\;=\;H\,\psi\;,$$

Since in this case, time reversal corresponds to $t \longrightarrow -\,t$ and $\psi \longrightarrow \psi^*$ (complex conjugation)

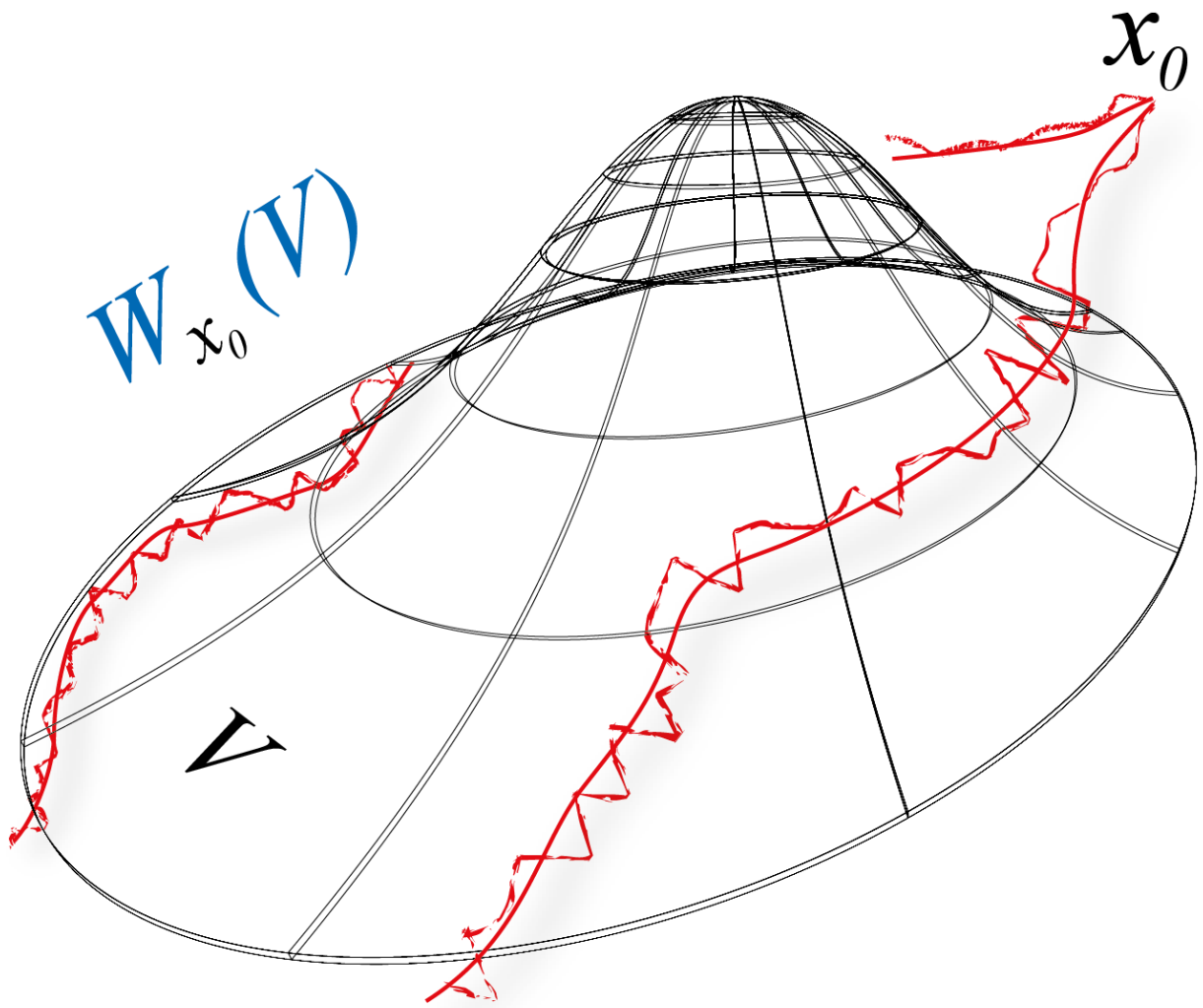


https://www.youtube.com/watch?v=2mIk3wBJDgE

However, time–reversal invariance does fail if we add to the Schrödinger equation the prescription (a non–linear stochastic trasformation) of how the *quantum state reduction* takes place when a measurement is performed.

The quantum state reduction procedure appears necessary in order to tackle with the measurement problem in QM, however its status as a foundational imput of QM is still much debated. There are those who strongly consider state reduction as a deep aspect of QM, explaining Nature's Time-reversal asymmetry. There are others who consider state reduction as a rather unsatisfactory procedure, at least as it stands, and that QM retains time-reversal symmetry at a fundamental level.

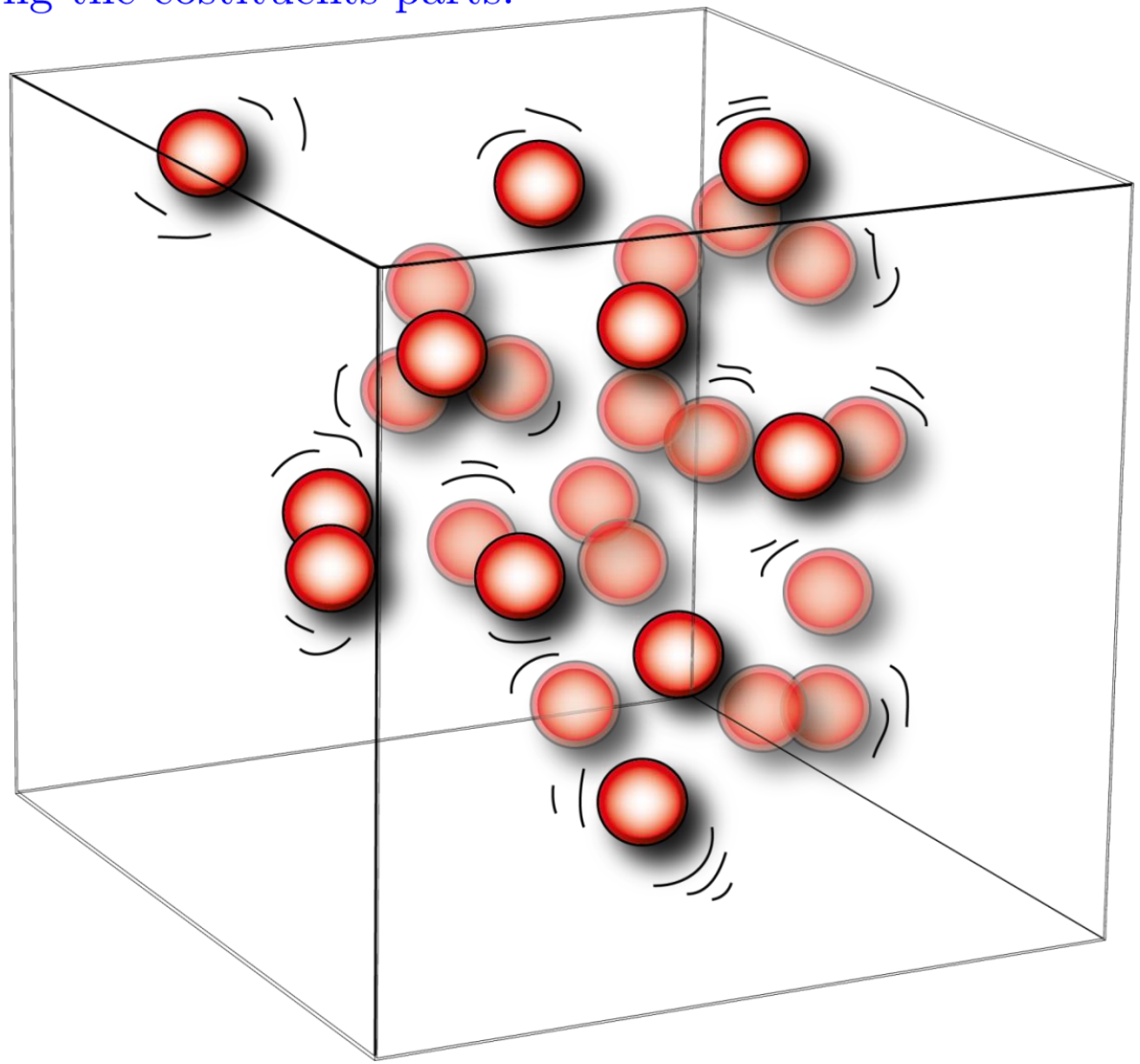
Quantum
Measure



An example of a deep interplay between "fundamental phsics" (here the quantum mechanics of a particle) and "complex physics" (here the equilirium statistical mechanics of random paths: Brownian motion). The evolution (Time!!!) of a quantum particle (a point, a 0-dimensional object) *vs.* the equilibrium (hence Time is not involved) statistical mechanics of paths (1-dimensional objects).

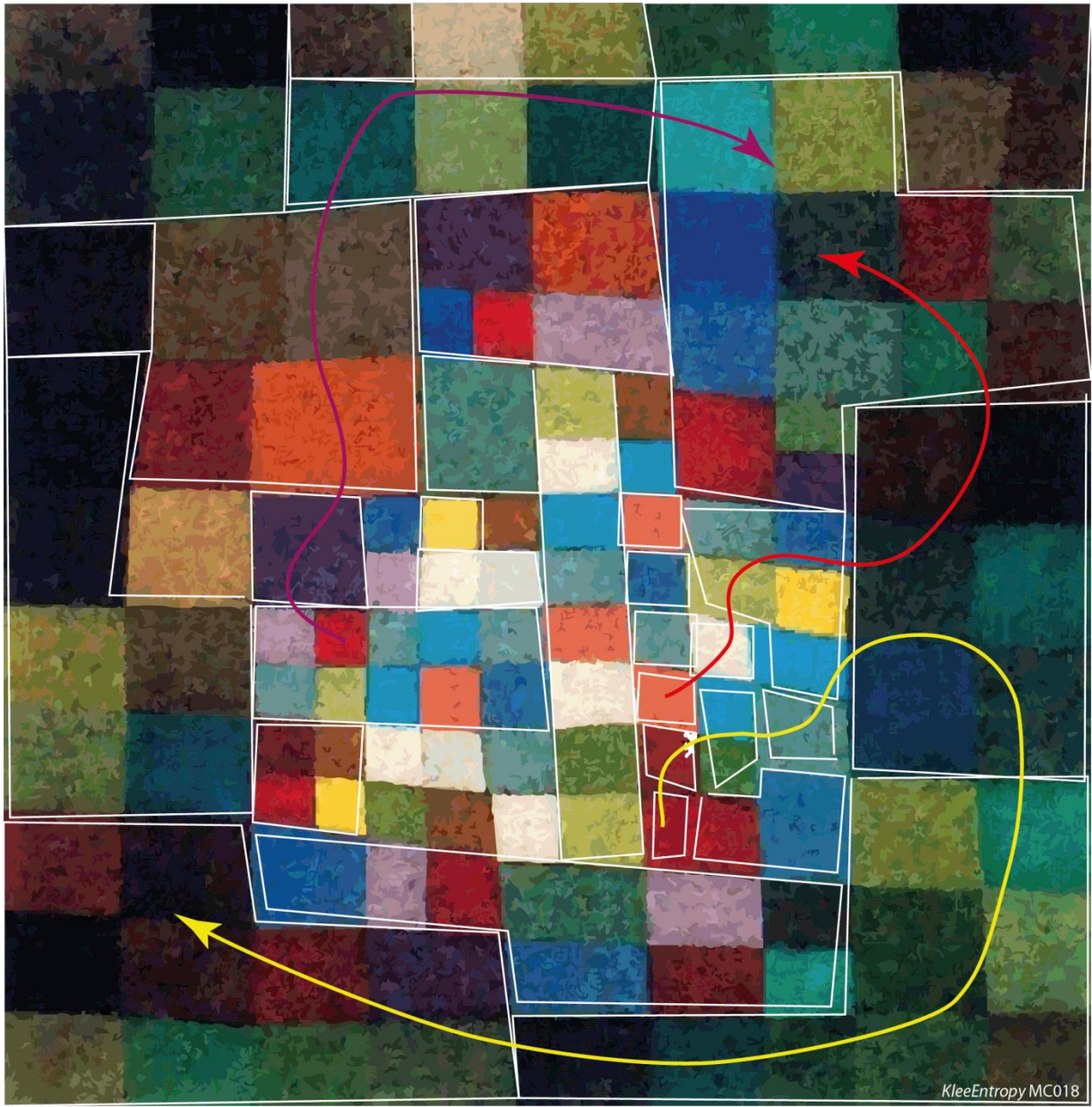
Time-reversal invariance does not preclude the onset of thermodynamical irreversibility and of a corresponding *Arrow of Time*. This sort of irreversibility is associated with the behavior of macroscopic collective variables which statistically average (over many possible distinct dynamical states) the local (time-reversible) interactions among the costituents parts.

The basic point is that as the number of degrees of freedom of the system becomes very large and we consider sufficiently large time scales, the collective variables follows a dynamical evolution of their own.



This is the *emergent point of view*, in which entropic mechanisms and the second law of thermodynamics play a key role.

- Boltzmann's Entropy
  $S = k_B \ln W$
- k_B
  $= 1.38065 \times 10^{-23}\, J/K$,
  the Boltzmann constant
- W = the number
  of real microstates
  corresponding to
  the gas macrostate



Both in the reductionist as well as in the emergent point of view, we have to put Time in relation with Space, sometimes in a very sophisticated and unexpected way. This gives rise to the kinematical (nowadays, dynamical) arena of physical laws: *Spacetime*. Its geometrical nature reflects the evolution of our attempts, over the centuries, to understand and model physical phenomena in a coherent framework.

& alors x ſera un nombre, & on n'aura que faire de diviſion. Cette maniere de conſidérer les quantités de plus de *trois dimenſions*, eſt auſſi exacte que l'autre; car les lettres algébriques peuvent toûjours être regardées comme repréſentant des nombres, rationels ou non. J'ai dit plus haut qu'il n'étoit pas poſſible de concevoir plus de trois *dimenſions*. Un homme d'eſprit de ma connoiſſance croit qu'on pourroit cependant regarder la durée comme une quatrieme *dimenſion*, & que le produit du tems par la ſolidité ſeroit en quelque maniere un produit de quatre *dimenſions*; cette idée peut être conteſtée, mais elle a, ce me ſemble, quelque mérite, quand ce ne ſeroit que celui de la nouveauté.

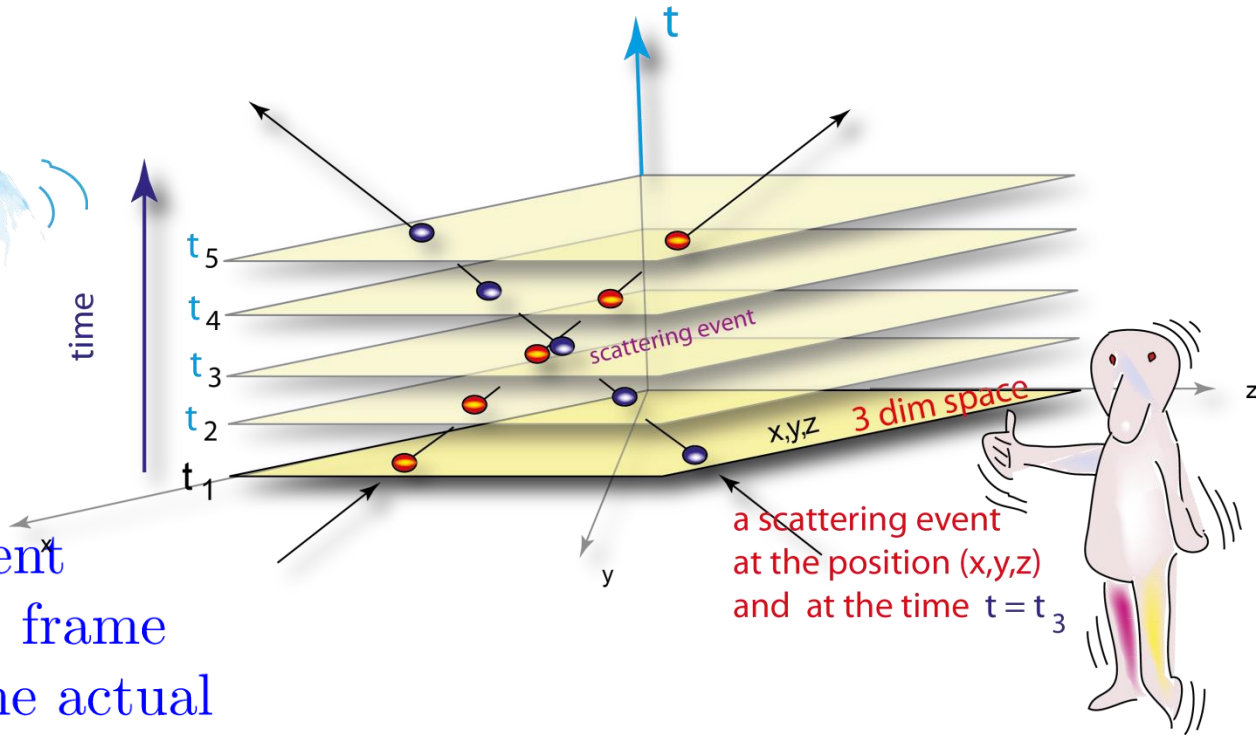
Voce: DIMENSION, (f.f. (Physique & Géometrié)
Jean Baptiste Le Rond d' Alembert,
Vol. 4, pp. 1010, (1754)

ENCYCLOPÉDIE
ou dictionnaire raisonné
des sciences, des arts et des métiers
Denis Diderot & Jean le Rond d'Alembert

Spacetime: the collection of all possible events.

We need four numbers in order to characterize an event with respect to a family of observers (frame of reference)

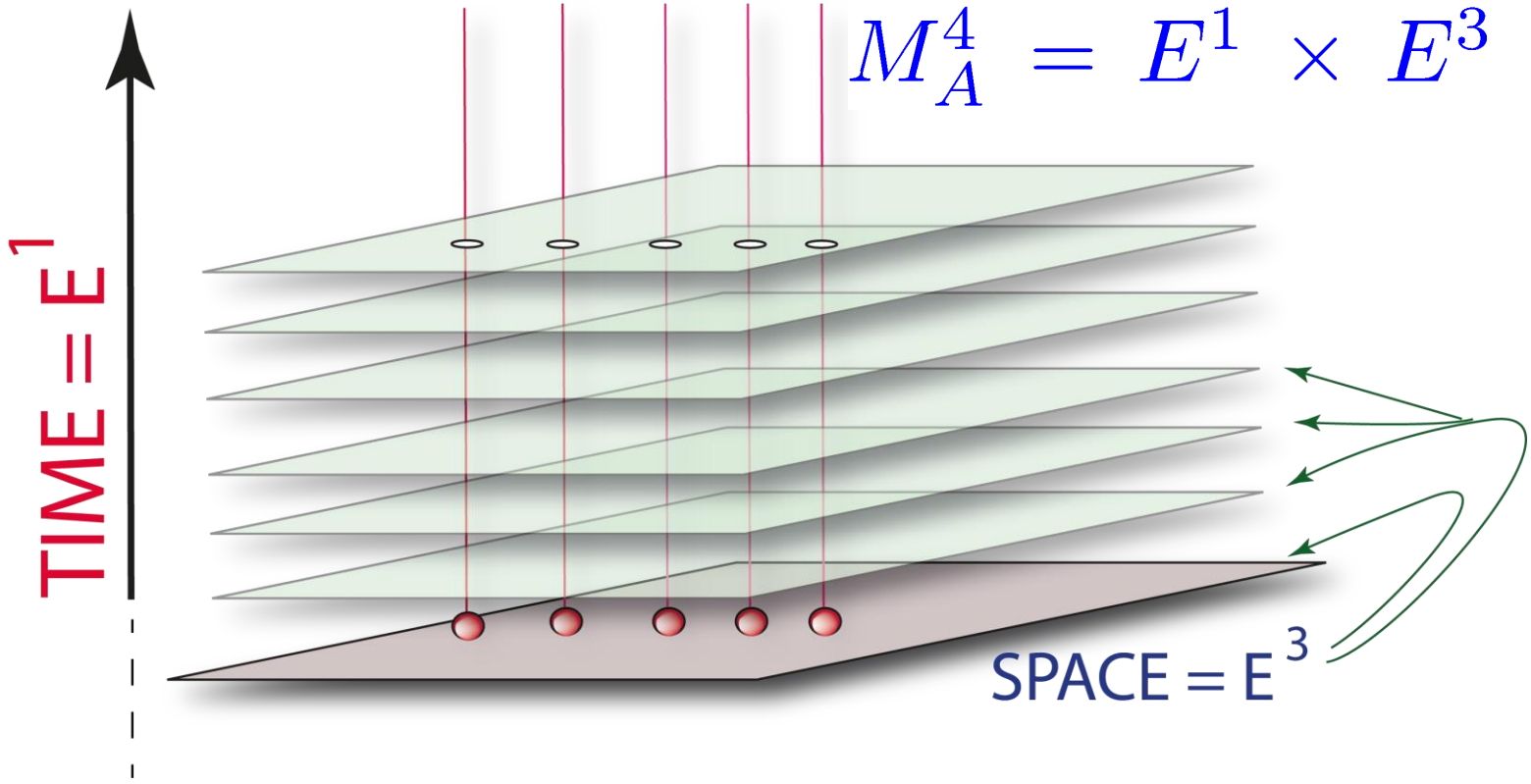
$(x, y, z; t)$: *where* $= (x, y, z)$ and *when* $= t$ an event occurs.



the occurrence of an event is independent from the frame of reference and from the actual coordinates used to represent it.

According to Aristotelian "Physics" the state of rest is dynamically preferred as compared with other states of motion. This implies the following view of

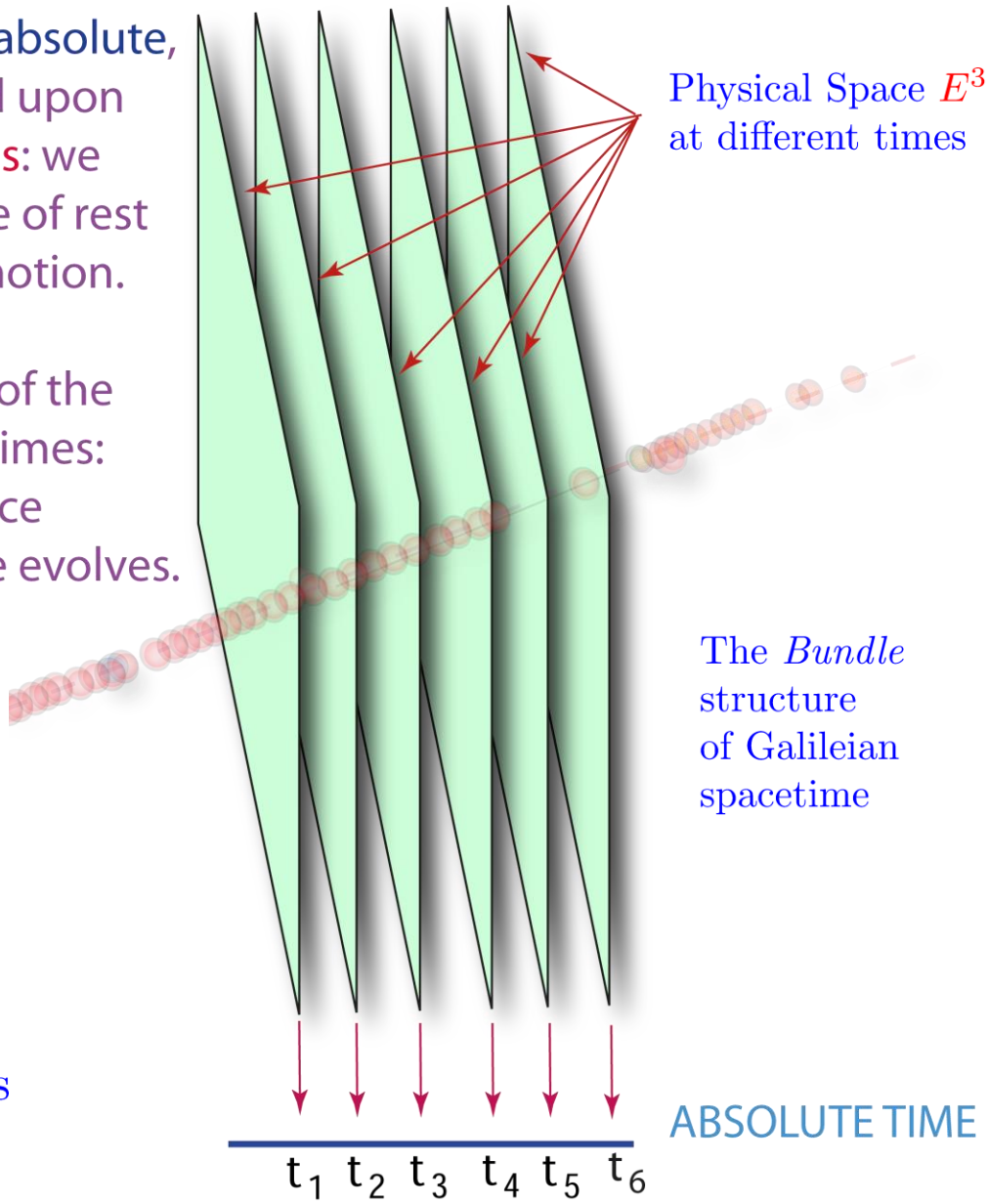
Aristotelian Spacetime



where E^3 is Euclidean 3-space where we measure distaces among points, and E^1 is a Euclidean 1-dimensional space along which we measure time intervals, (no preferred origin of time). Aristotelian Spacetime is a totally static arena for physics. (See R. Penrose's *The Road to Reality* Chap. XVII for a nice geometrical introduction).

Link 2

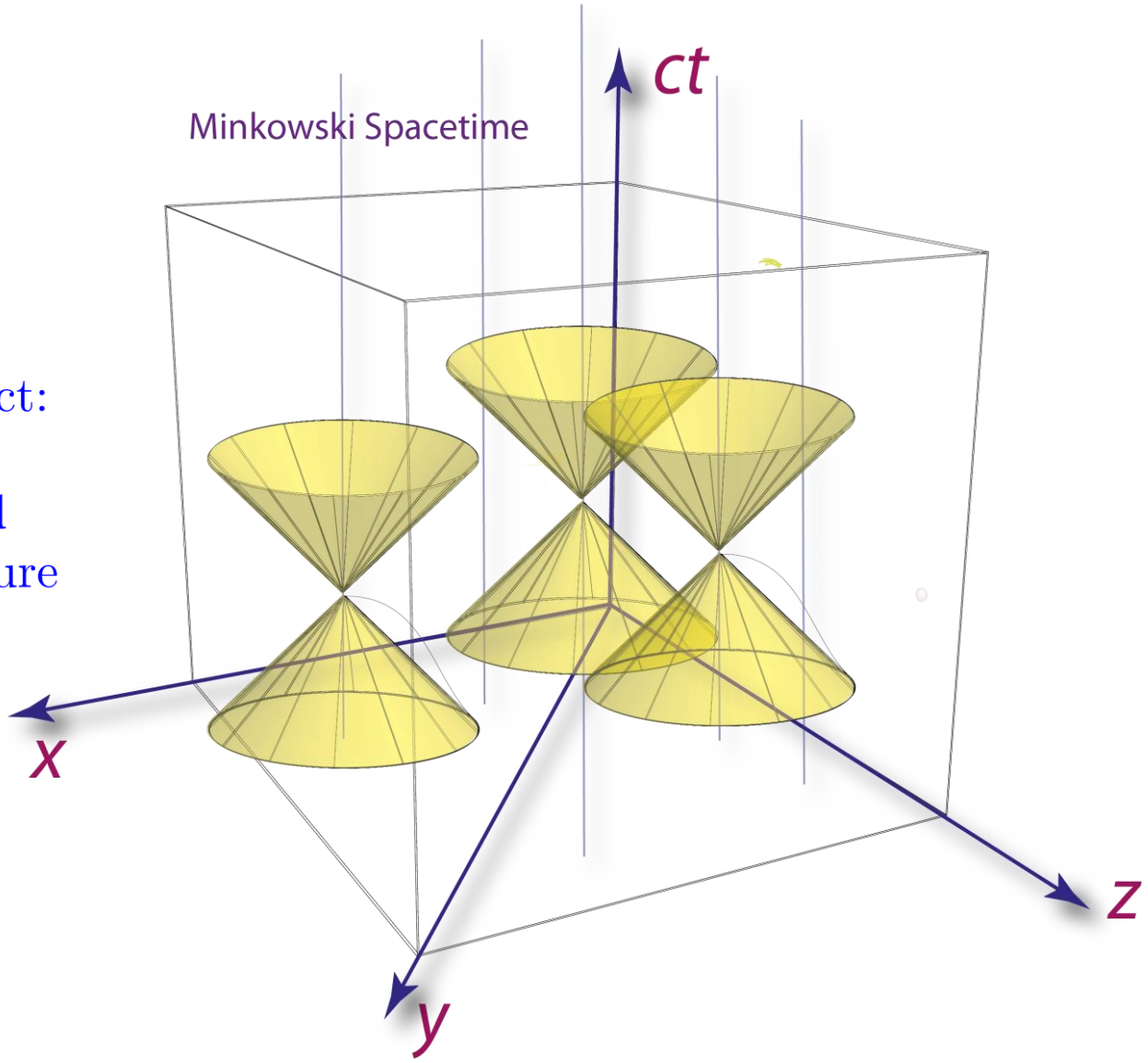
In Galileian physics Time is absolute, Space is not since it is acted upon by Galileian Transformations: we cannot distinguish the state of rest from the state of uniform motion. Hence, there is no natural identification of the points of the physical space at different times: there is no background space which remains fixed as time evolves.



- Galileian Spacetime M^4_G

is not a product of 3-space and time, It is a collection, parametrized by absolute time, of different copies of the physical space E^3. Each (Galileian) spacetime event is naturally assigned a time, but there is no natural assignment of a spatial location as a geometrial point of an absolute E^3, (the spatial location depends on the frame we are using) .

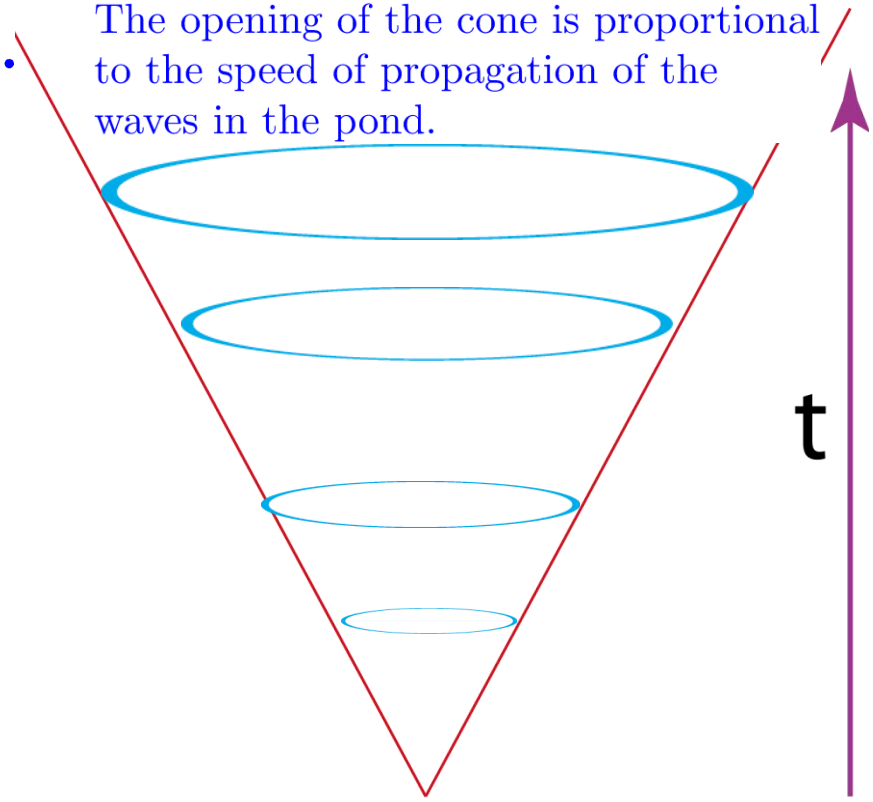
In (Special) Relativistic Physics, spacetime is acted upon by *Poincaré Transformations*. Time is no longer absolute. Minkowski Spacetime M^4 is a new geometrical object: a 4–dimensional manifold ($\simeq \mathbb{R}^4$) endowed with a geometrical structure characterizing the causal relations among events. No event is privileged.

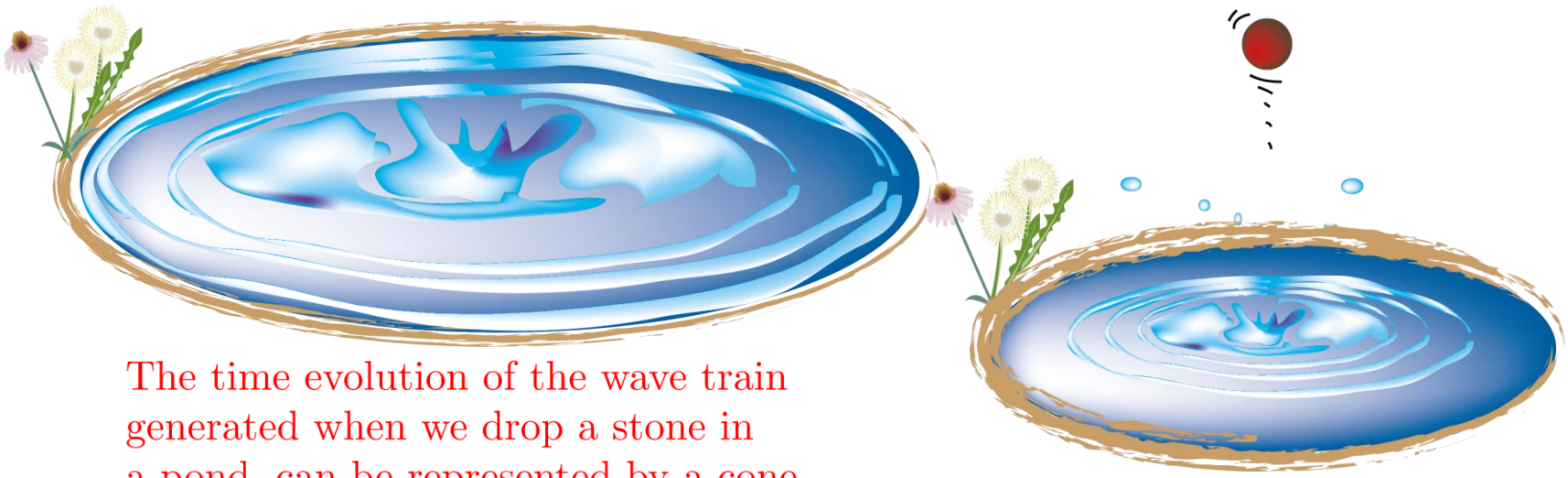


Link 3 Link 4

Causal Structure $\Longleftrightarrow$ The Light (Null) cone

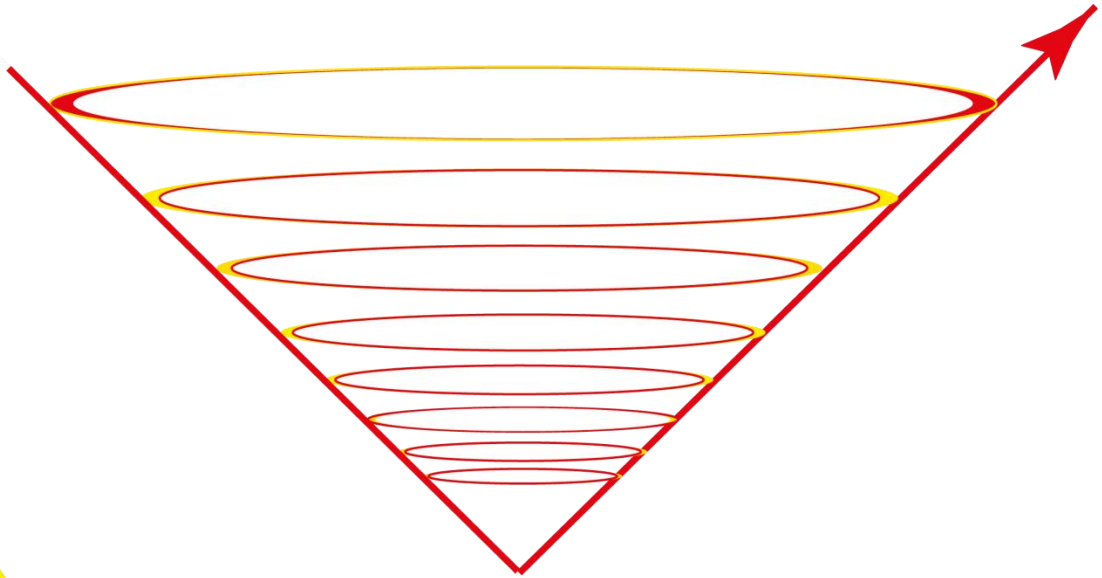
... Light Cone heuristics...

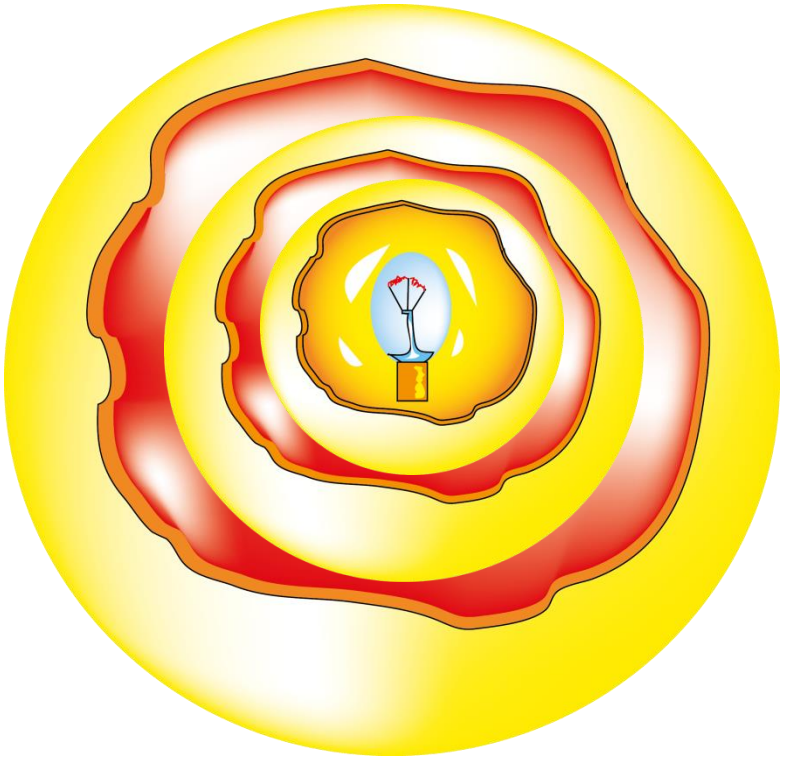


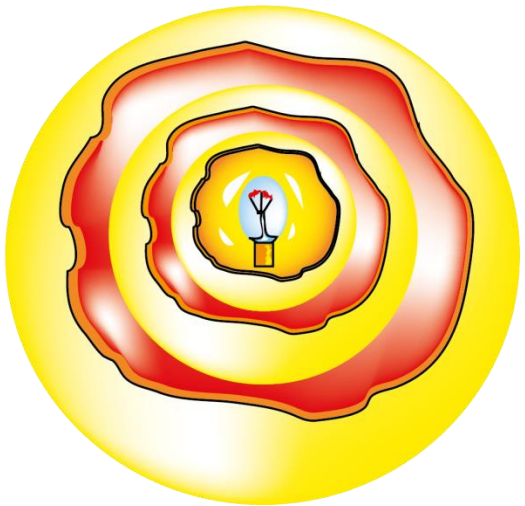


The time evolution of the wave train generated when we drop a stone in a pond, can be represented by a cone.

Triggering a light flash in spacetime is like trowing a stone in a pond.The spacetime history of the wavefronts of a light flash: The Light Cone. The opening of the cone is the speed of light in vacuum.

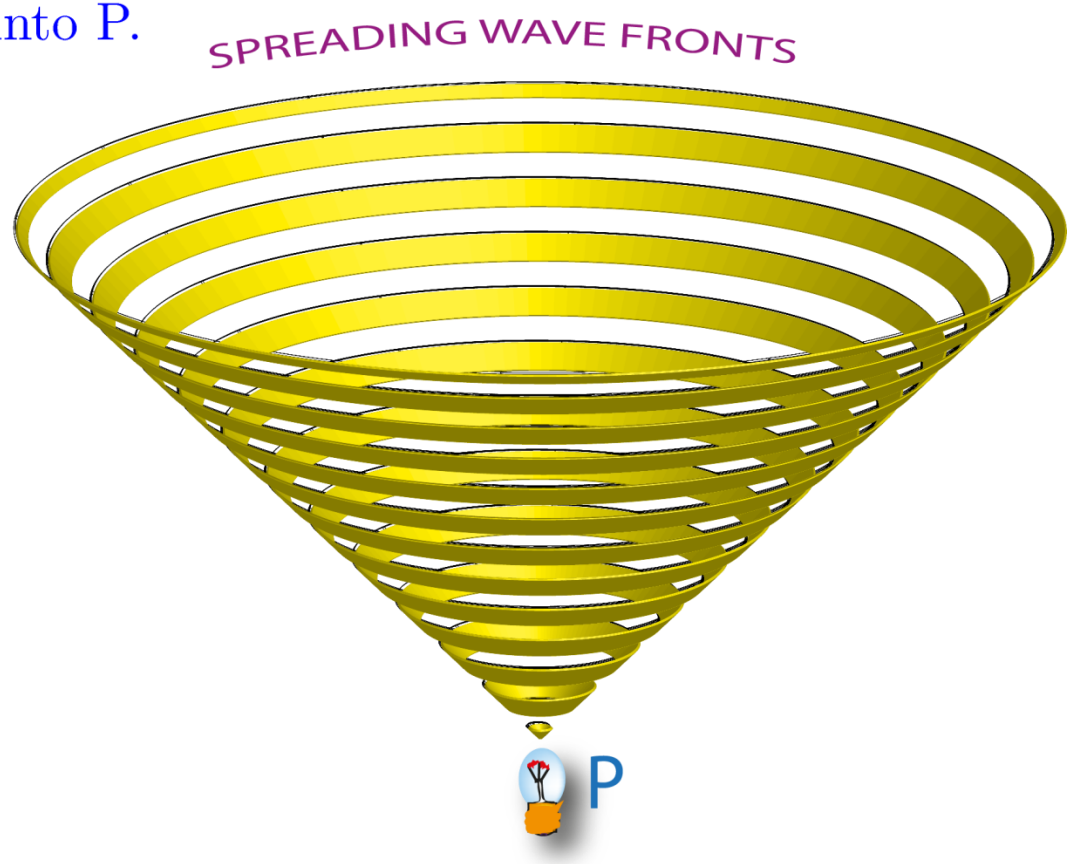






celeritas = 299,792,458 metres per second

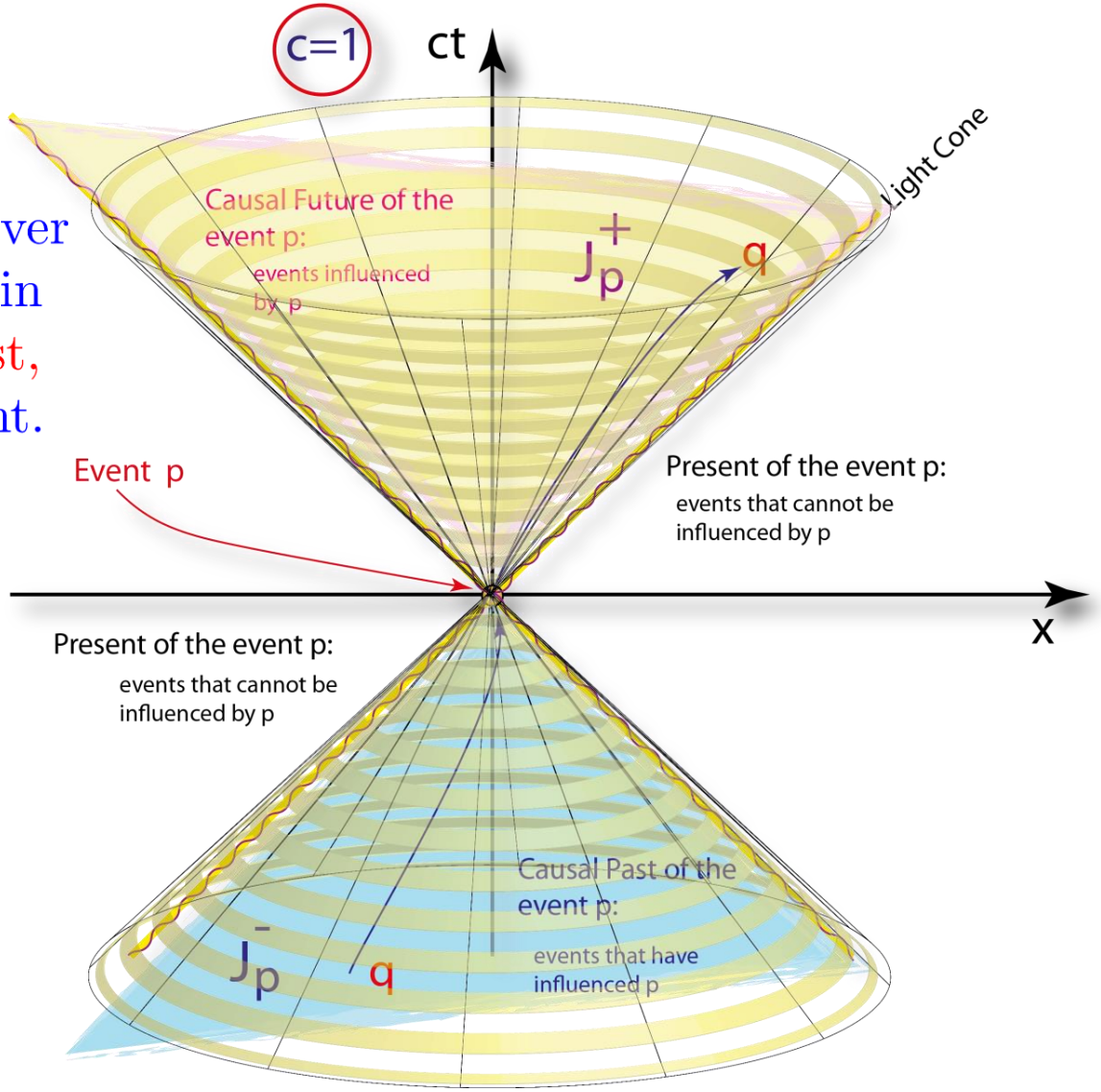
The future light cone at an event P may be thought of as the world-history of the propagation of a light-front emitted (in vacuum) by a light-flash at P. Each light-front is a 2-sphere expanding from P. The past light cone at P is the world history of a light front converging into P.



The light cone is not peculiar to electromagnetism. The propagation of every massless field is characterized by this cone. The light-cone is a *property* of Minkowski spacetime, related to its geometric properties.

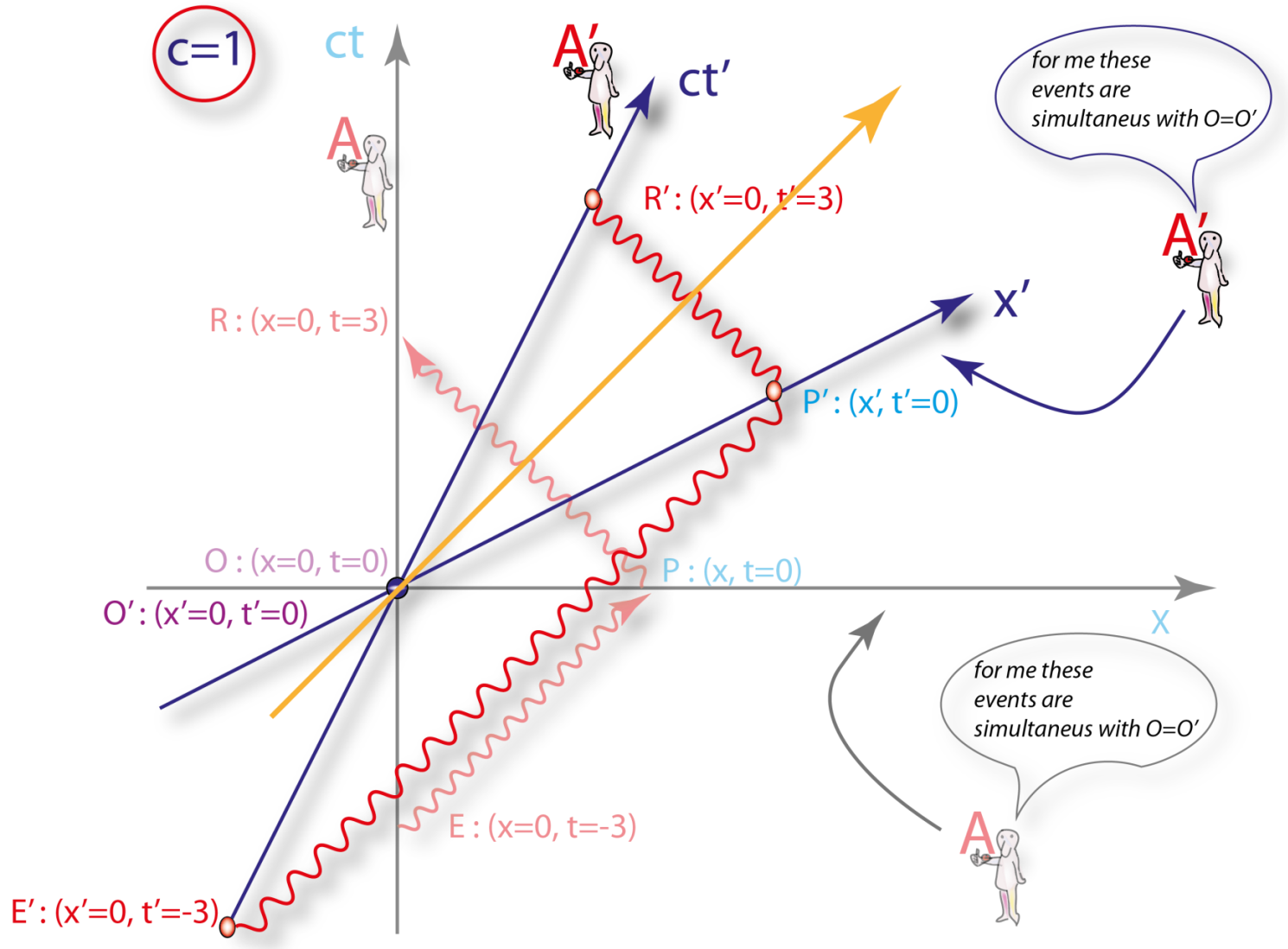
The *causality principle* together with special relativity implies that no physical signal can travel faster than light. It follows that **the light cone determines the causal structure**: how spacetime events are causally related to each other.

Simultaneity of events is observer dependent: deep modification in the characterization of the Past, Present, and Future of an event.

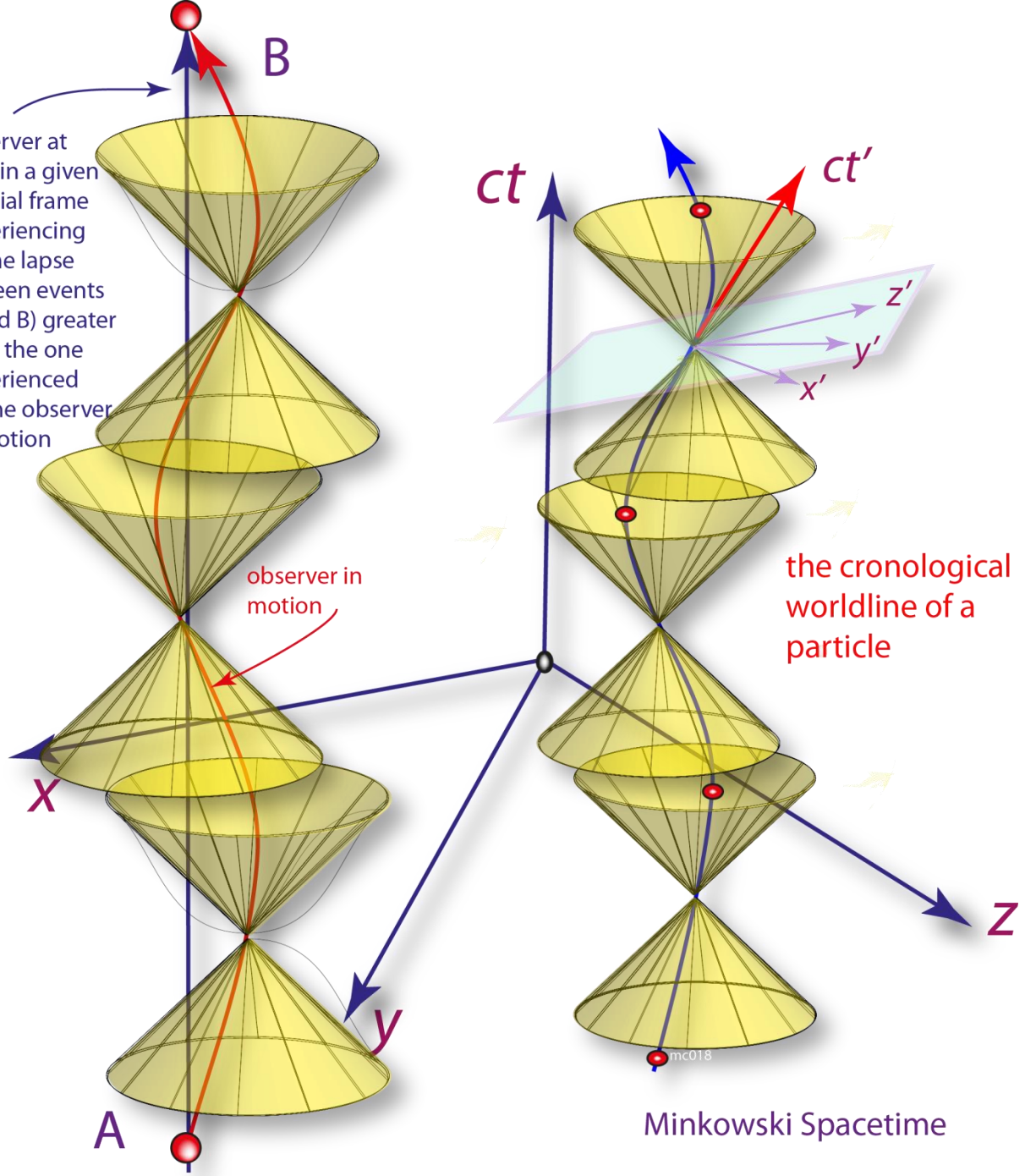


"What *is* the Presents?"

Presents because there are infinitely many 3-dimensional spaces that can be interpreted as the *Present*($\mathcal{O}$) of a given event $\mathcal{O}$: What is the *Present* is an observer–dependent notion!



The *proper time* along the worldline of a particle depends on its velocity. The time interval between two events depends on the state of motion of the observer.



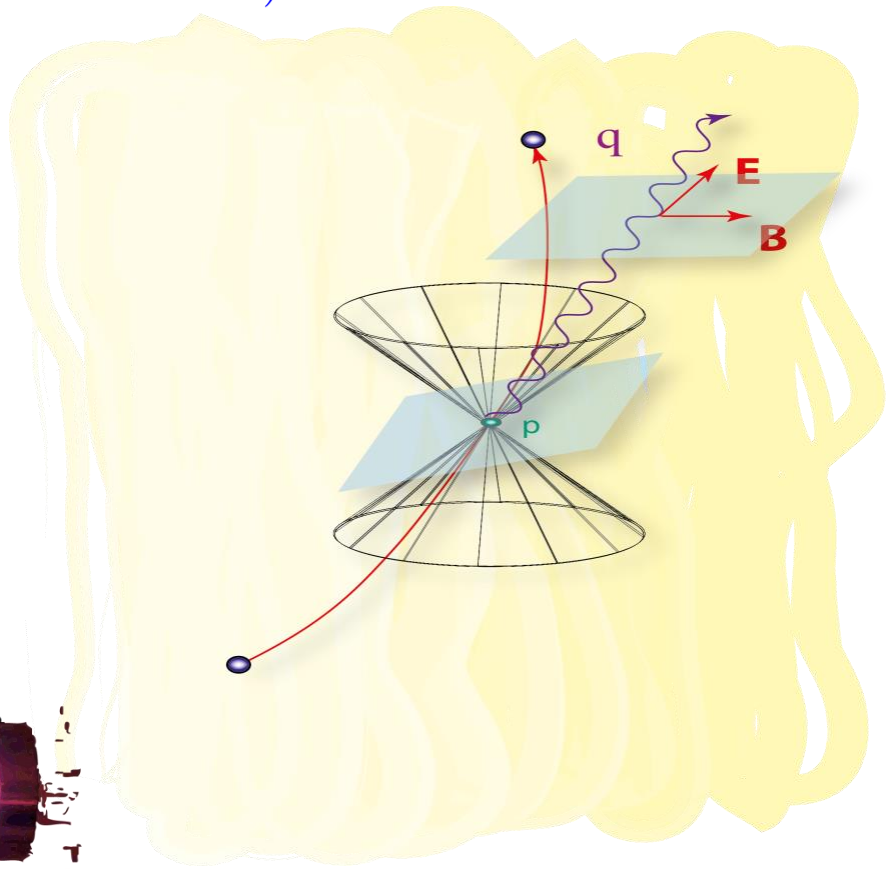
Minkowski spacetime is the appropriate starting point for discussing the complex grammar of time in modern physics.

The central aspect of Minkowski spacetime is the fact that the speed of light in vacuum is frame–independent. There two experimental measure which reach an incredible level of accuracy and which play a basic role for what concerns the characterization of Physical Time. These are the determination of the speed of light in vacuum c and of the Rydberg constant R_∞ (related to the emission spectra produced when atoms of a dilute gas are excited).

Link 5

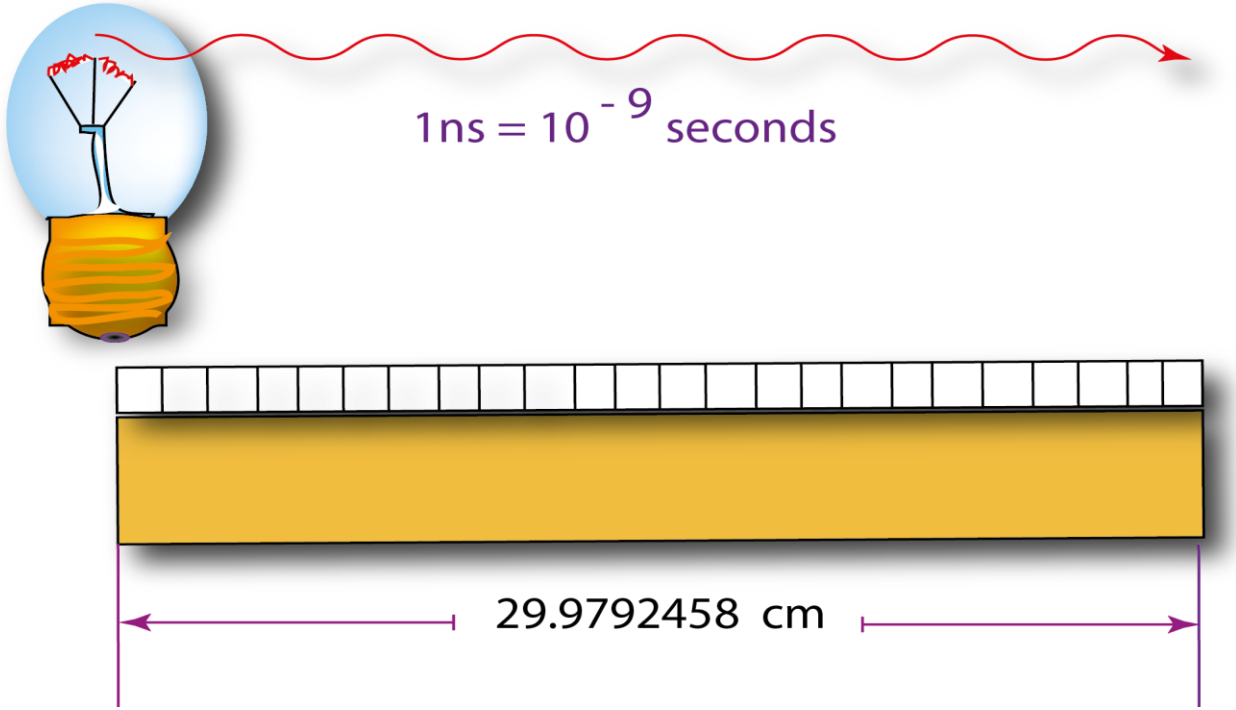
- $c = 2.99792458 \times 10^{10}\, cm/s$

- $R_\infty := \frac{m_e\, e^4}{8\epsilon_0\, h^3\, c}$
  $= 10\,973\,731,568\,508(65)\, m^{-1}$



The constancy of the speed of light has been verified with an extreme accuracy, to the effect that its constancy is now used to REDEFINE the centimeter in terms of the second:

- 1 centimeter := the distance traveled in vacuum by light in $(1/2.99792458) \times 10^{-10}$ seconds.

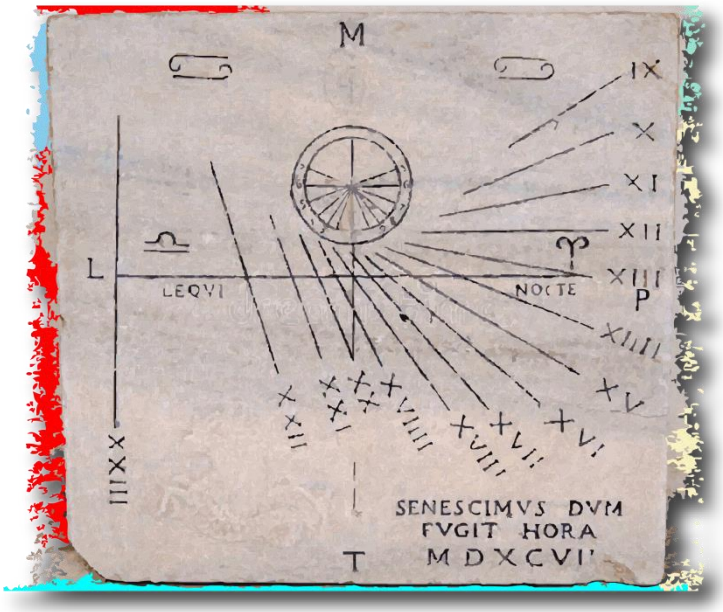


Perhaps easier to redefine, in the *Imperial Units*, the

- 1 ft := the distance traveled in vacuum by light in 10^{-9} seconds $= 1\,ns$.

Hence, measures of spatial distances are measures of time!

Time is measured by clocks, *i.e.* by a physical system which exhibits a periodic motion: A sundial, characterized by an astronomical day-period; A pendulum, the period of which is determined by the length of the pendulum rod ... . In both these examples, (and in many more), the period is not determined a priori and depends from many conditions. For instance we may have the secular slowing down of the Earth rotation; or the dilation/contraction in the length of the pendulum rod due to temperature variations.



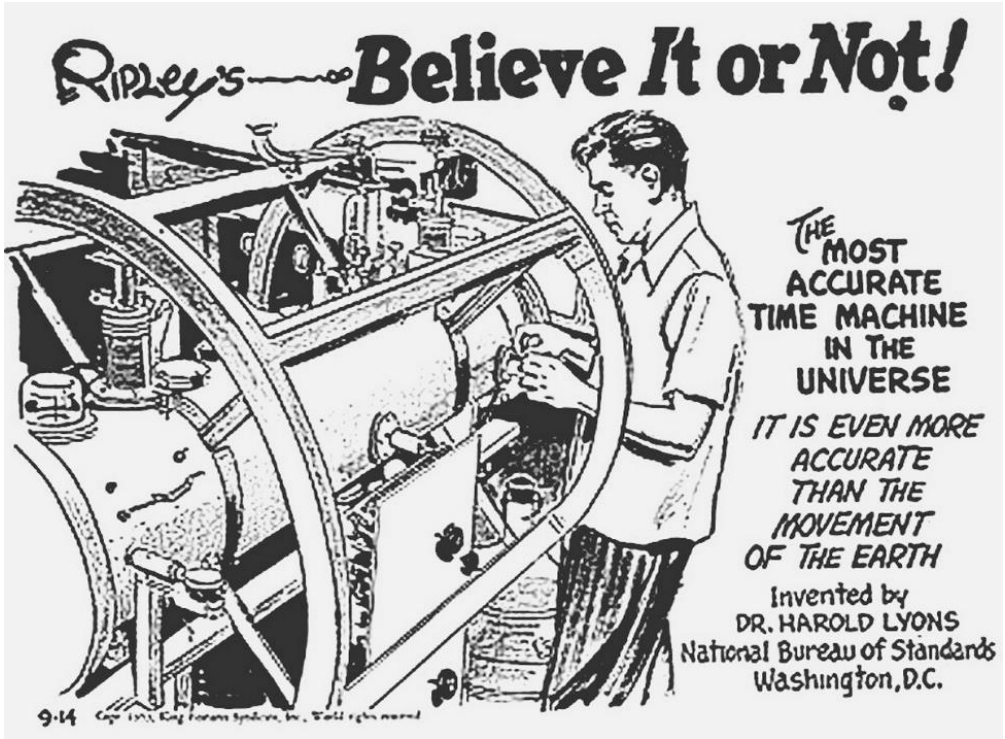
a *klepshydra*, (et. *stealing* water from cisterns).

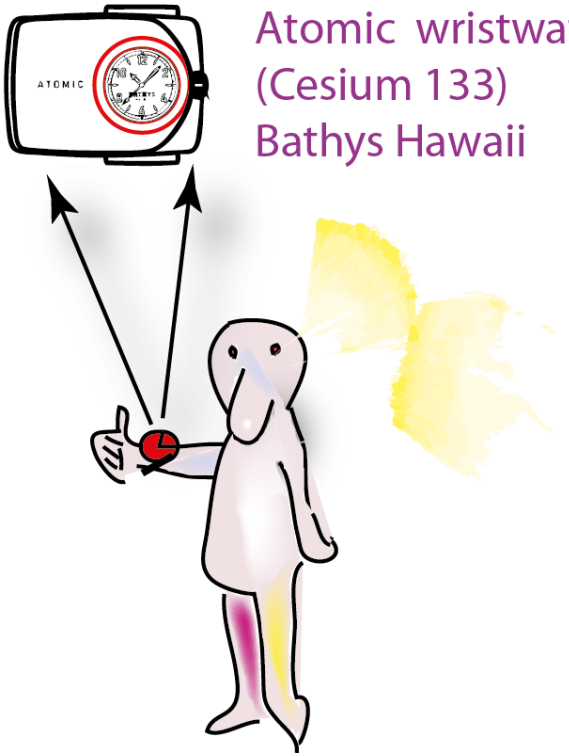
These systems are not really able to provide an exact unit of time which is defined a priori.

- But now we have Atomic clocks: astonishingly reliable because the Rydberg constant, playing a basic role in their function, is known to an incredible accuracy.

- The period of these clocks depends only from the fundamental constants of Nature, hence it is universal under suitable experimental conditions.

- We (IS) *DEFINE* the second as the duration of 9,192,631,770 cycles of the radiation corresponding to the transition between two energy levels of the caesium-133 atom at rest at a temperature of "absolute zero".





Atomic wristwatch
(Cesium 133)
Bathys Hawaii

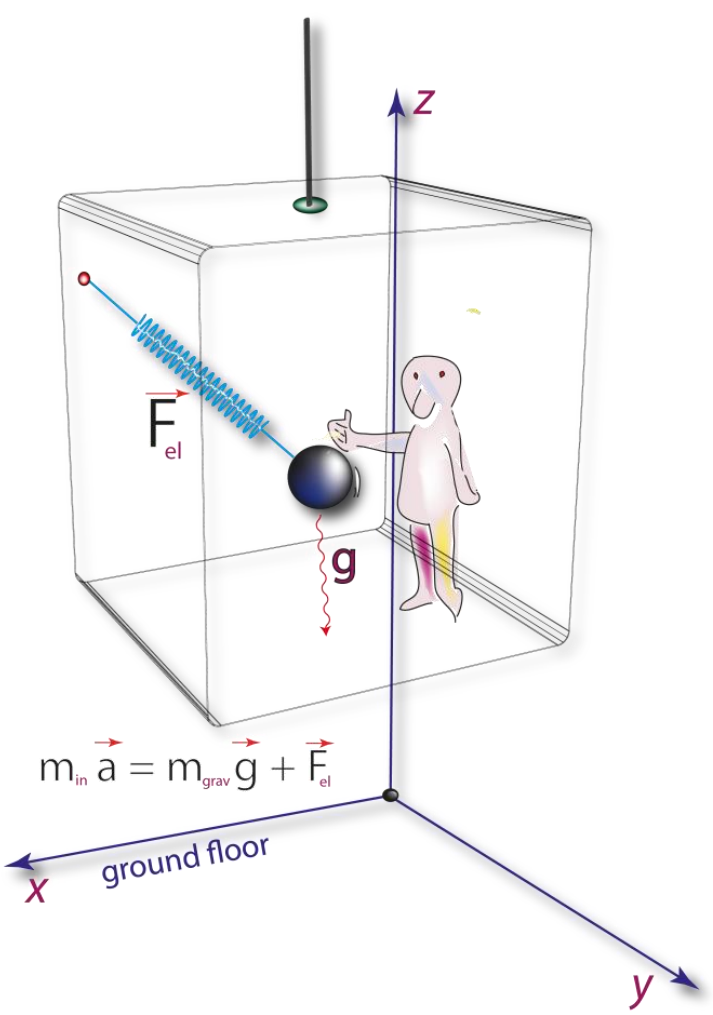
... But with Minkowski spacetime we do not cover the entire repertoire of possible spacetimes (and hence "times") in Physics ... because of GRAVITY ... the weakest among the known interactions, but the most evident because it is universal, hence its effects are cumulative.

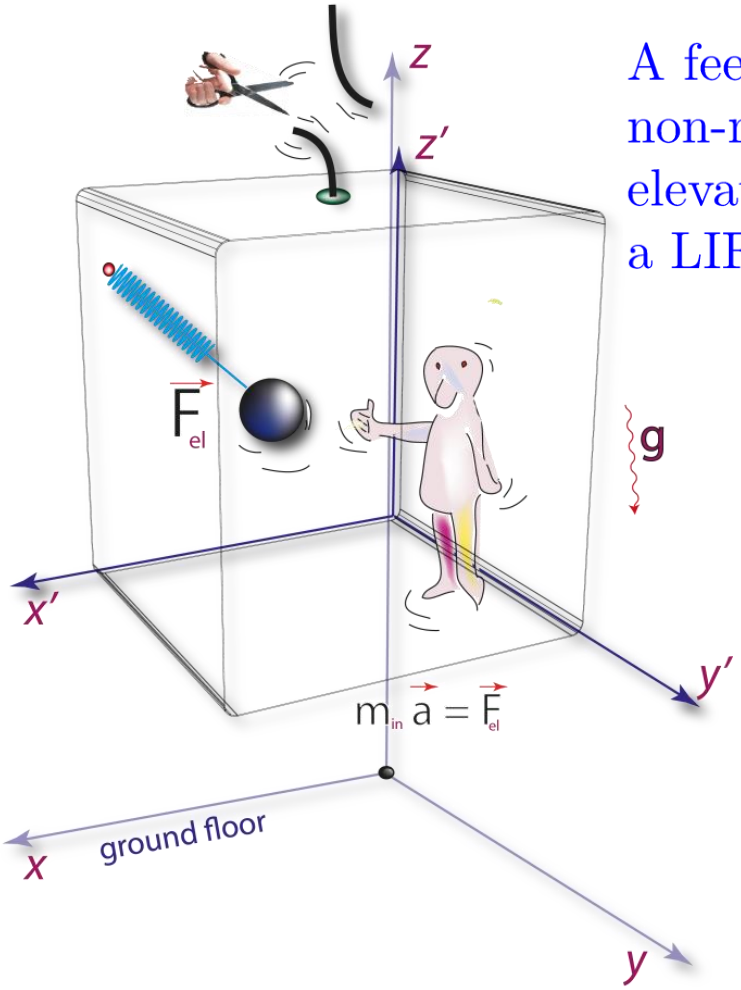
$$\vec{F}_{Newt} = \frac{G M_1 m_2}{r^3} \vec{r}$$

$$|F|_{Newt} = \frac{G M_1 m_2}{r^2}$$

Gravity is a force which affects every particle in the same way, as first recognized by Galileo.

This implies that the gravitational field is locally eliminable.

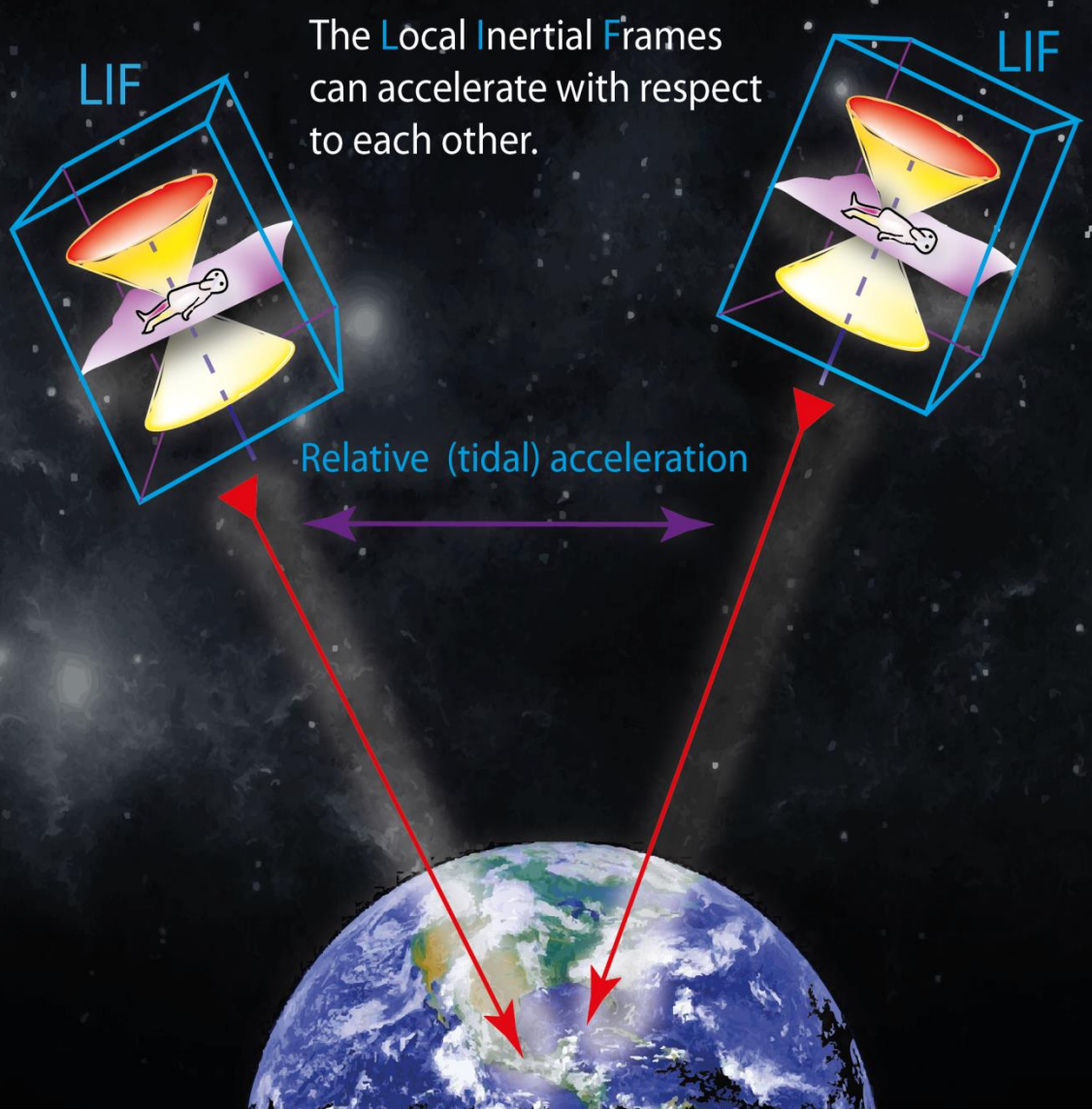




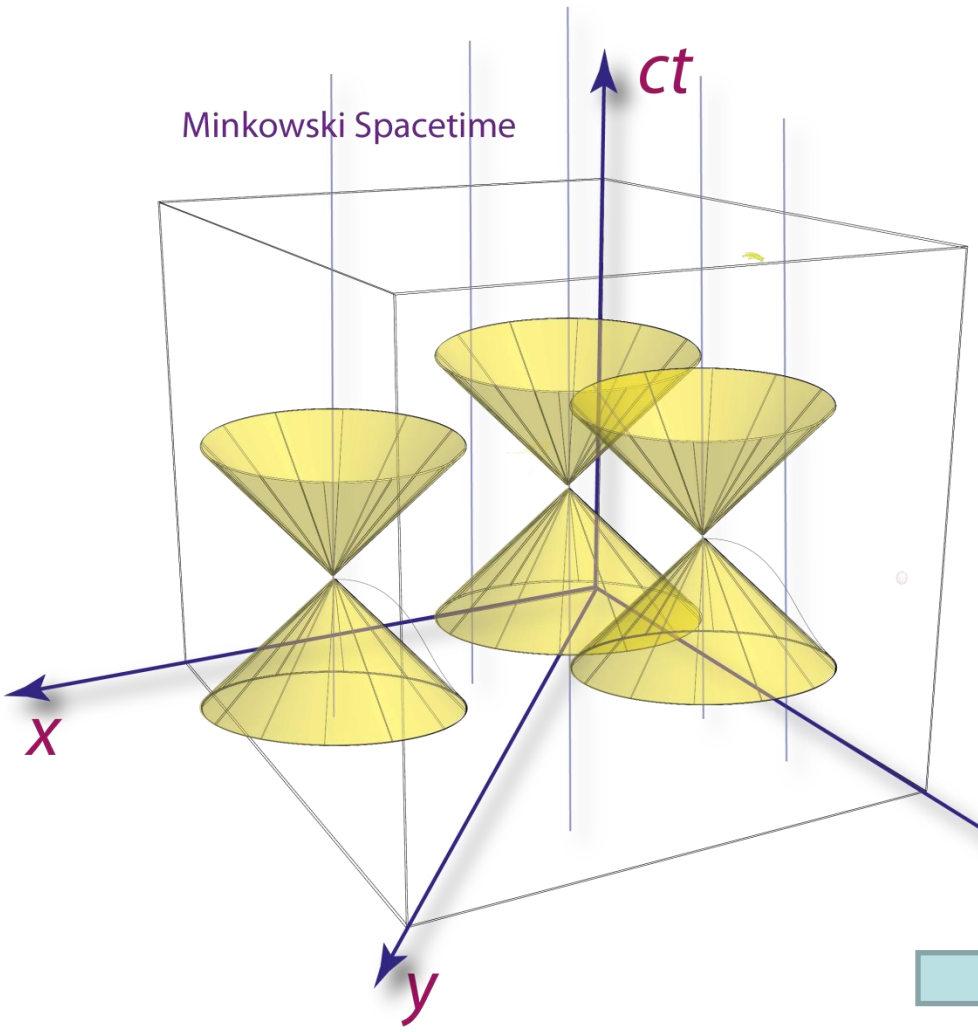
A feely-falling, non-rotating, elevator as a LIF

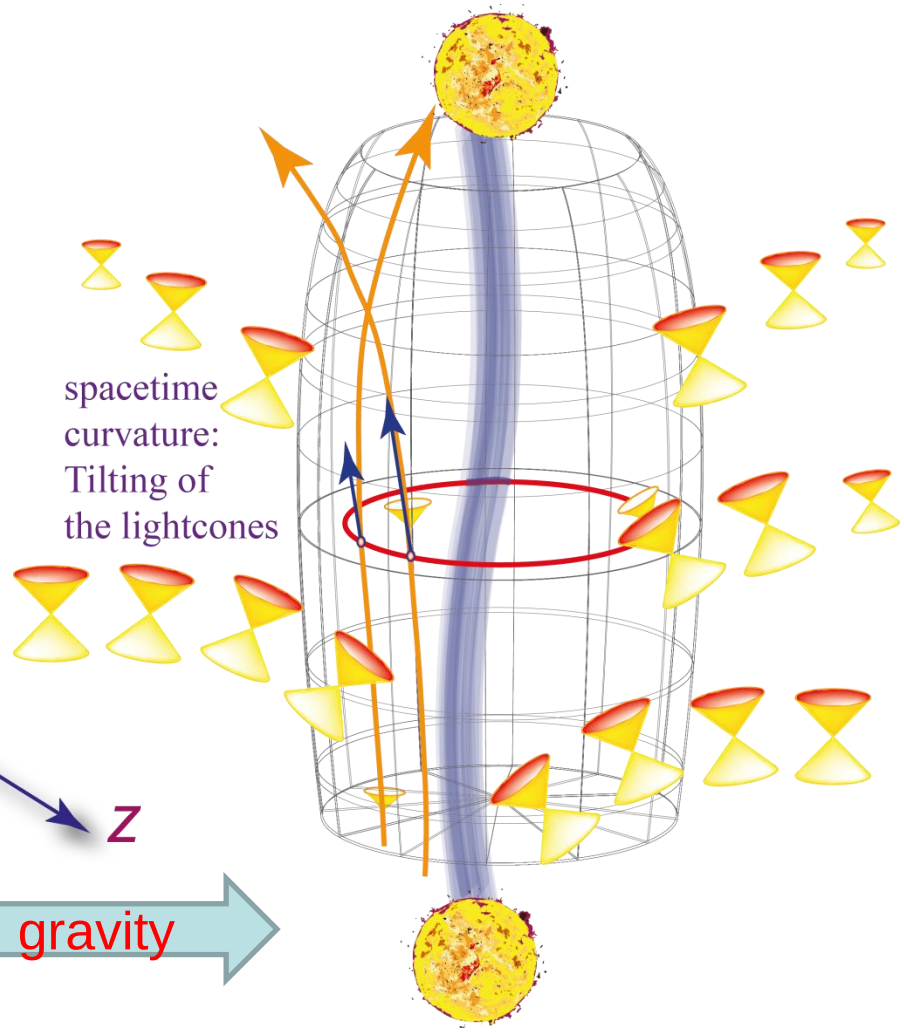
In a Local Inertial Frame (**LIF**) we experience the Physics as in Minkowski spacetime (Special Relativity): (Einstein's equivalence principle).

In special relativity the causal cones are all equivalent: we can map one into any other via (active) Poincare' trasformations. Hence, the causal structure of Minkowski spacetime is rigid. However, if we switch on gravity this is no longer true ...

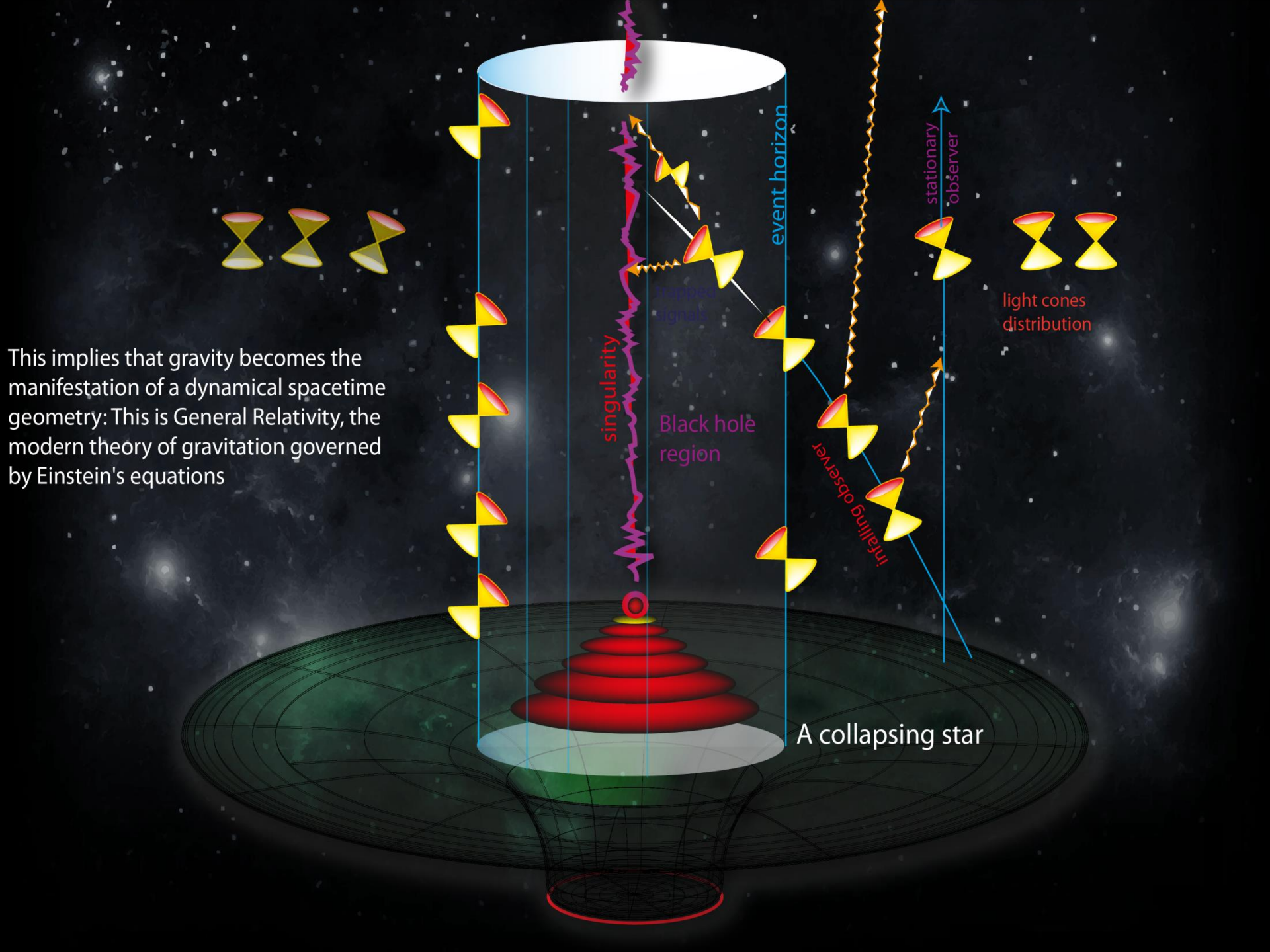


It follows that gravity deforms (and determines) the causal structure of space-time: Spacetime is causally not rigid and static: its causal properties react to all form of mass energy, and so does the (proper) time any observer experiences.





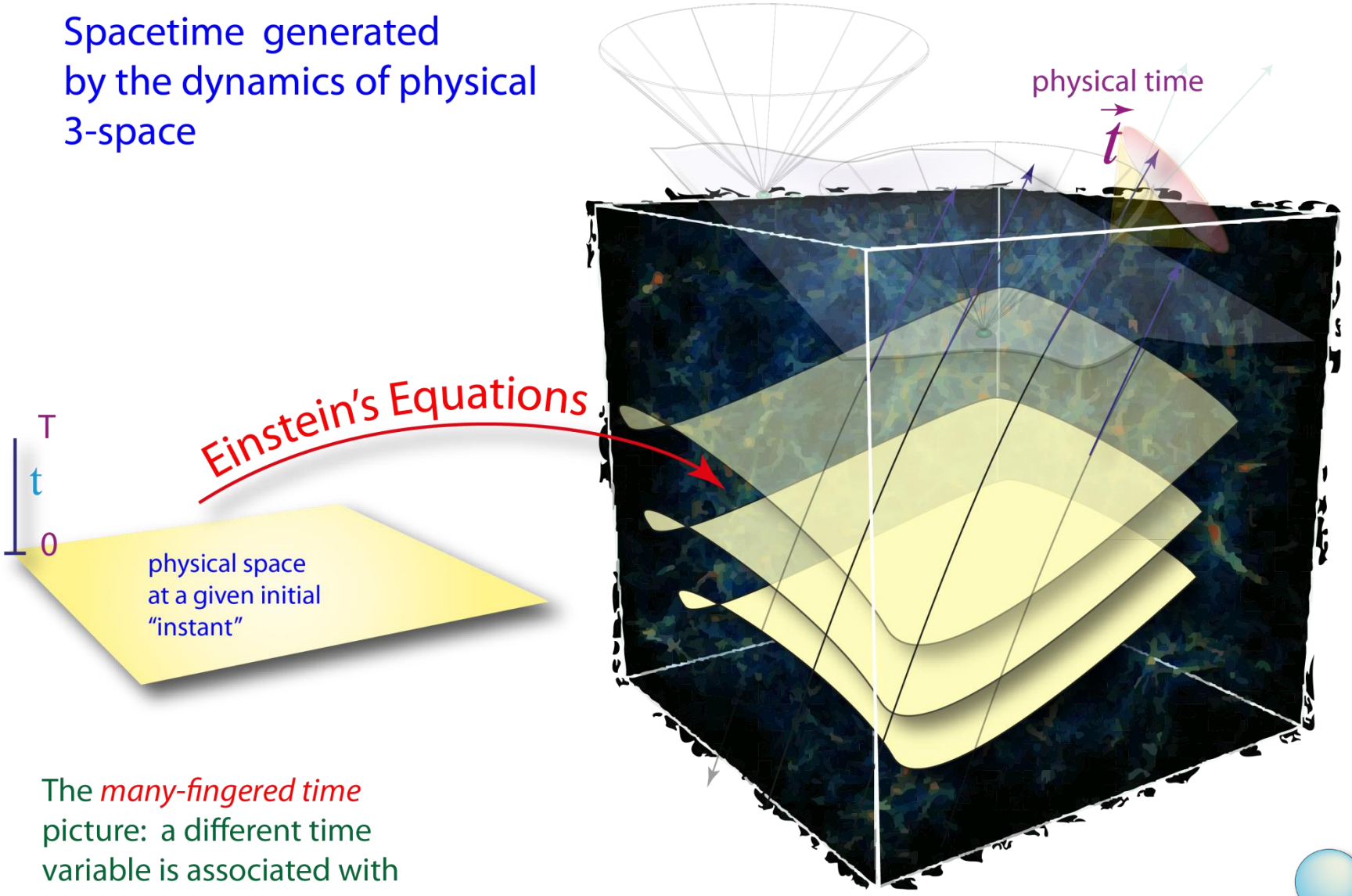
gravity



This implies that gravity becomes the manifestation of a dynamical spacetime geometry: This is General Relativity, the modern theory of gravitation governed by Einstein's equations

In the spacetime model of general relativity physical Time can be seen as generated through the dynamics of space.

Spacetime generated by the dynamics of physical 3-space



The *many-fingered time* picture: a different time variable is associated with each point in space

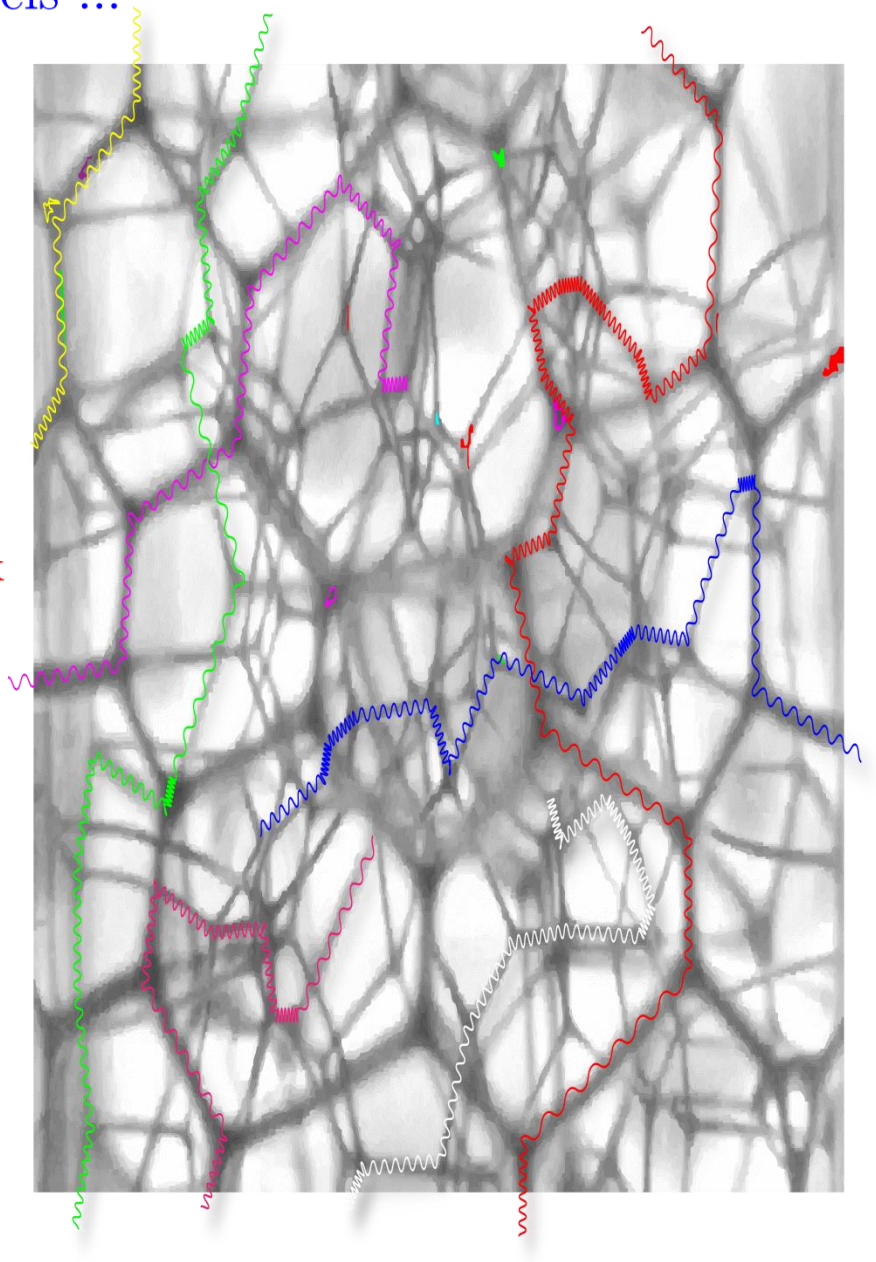
Link 6

Nice, ... but since we should never cease to intrigue philosophers ... we need to look for more sophisticated spacetime models ...

Newton's gravitational constant G, sets the fundamental (Planck) scale of time:
there is a unique combination of $\hbar$, c,
and G which has dimension of time: the
Planck time $T_{planck} = \sqrt{\hbar G c^{-5}} \approx 10^{-43} s$

There are reasons to believe that, associated with this last of the dimensional parameters of nature, there is a new conceptual change in the framework of Physics: Quantum Gravity.

Tested high–energy physics occurs at
time–scales $\gg T_P$
where the continuum picture works well.



- Will the continuum assumption break down at the Planck time?
- If so, the space–time continuum is a coarse–grained approximation? of what?
- Is Time quantized?

Insights and Remarks
Hyperlinked Slides
(Some in Italian)

Link 1 

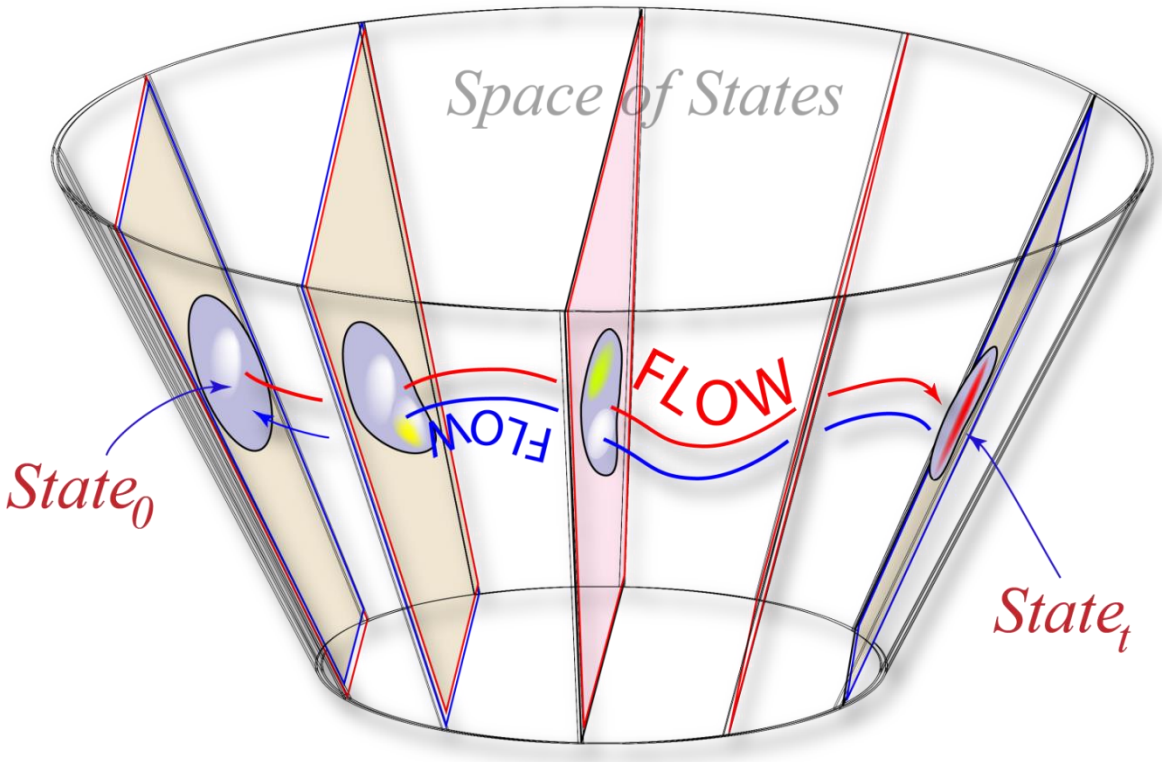
Time-Reversal transformation (dynamical systems)

Explicitly, if the (time)-forward dynamics of our system is described by an invertible *flow* Φ_t

$$\text{State}_t \;=\; \Phi_t\,(\text{State}_0)$$

in the (phase) space $\{\text{State}_t\}$ of its possible states , then the system is dynamically reversible if there exists a trasformation Λ (with $\Lambda^2 \;=\; 1$) of states such that (T. Jacobs, C. Maes)

$$\Lambda\,\text{State}_t\,\Lambda \;=\; \text{State}_t^{-\,1}$$



Assume that at time $t = t_0$ we measure an observable $\mathcal{O}$ (represented by an operator $\widetilde{\mathcal{O}}$). Before the measurement occurs, *i.e.* for $t < t_0$, the wave function $\Psi(x)$ of the system under consideration is a linear combination of the eigenfunctions $\{\phi_k\}_{k\in\mathbb{N}}$ of the operator $\widetilde{\mathcal{O}}$

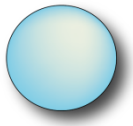
$$\Psi_{t<t_0}(x) = \sum_k a_k \, \phi_k(x) \, .$$

As soon as the measure is carried out the wave function reduces to some specific eigenfunction $\phi_h(x)$ of $\widetilde{\mathcal{O}}$

$$\Psi_{t>t_0}(x) = a_h \, \phi_h(x) \, .$$

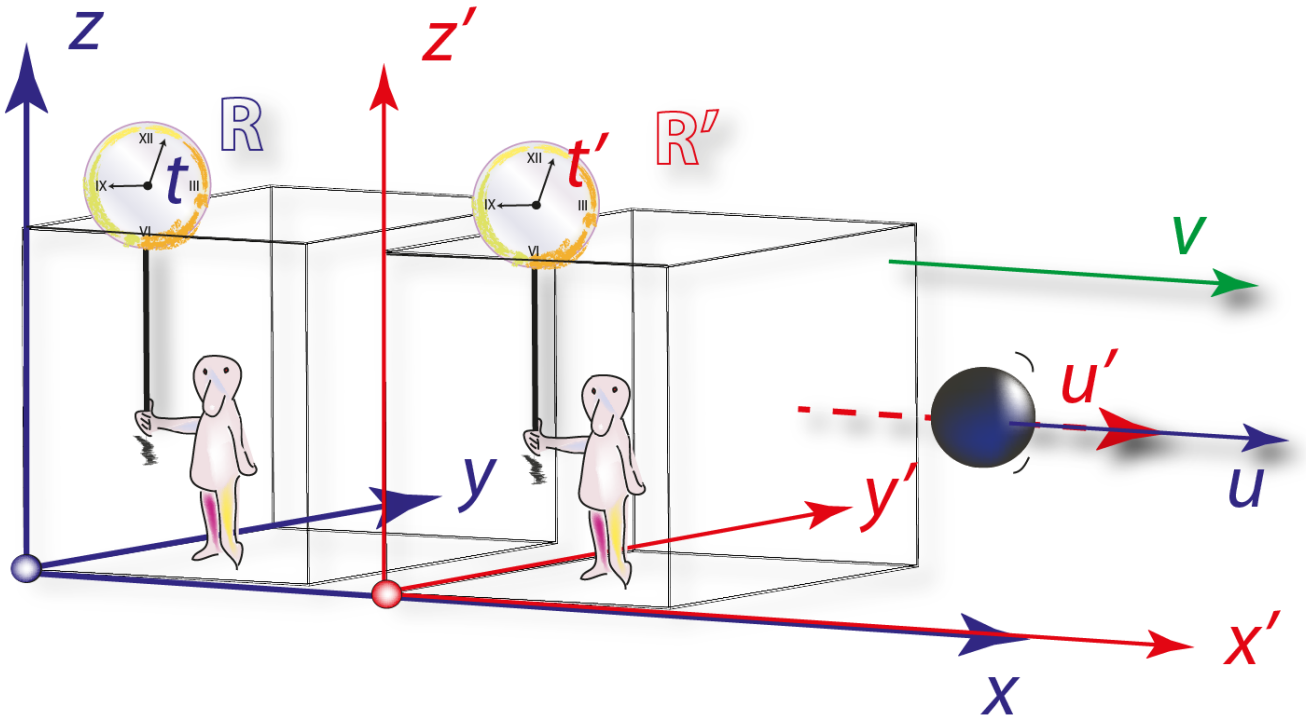
Note that:

- The specific eignefunction $\phi_h(x)$ (*i.e.* the index h) is not determined by the data on the system for $t < t_0$. These data just allow us to characterize the probability for $\phi_h(x)$
- Hence, a fundamental aspect of measurement in quantum theory implies that the knowledge of the initial state $\Psi_{t<t_0}(x)$ does not uniquely determine the final state $\Psi_{t>t_0}(x)$



Link 2  Galileian Transformations

R': Inertial Frame of Reference in uniform rectlinear motion (along the x–axis), with velocity V w.r.t. the inertial frame R.



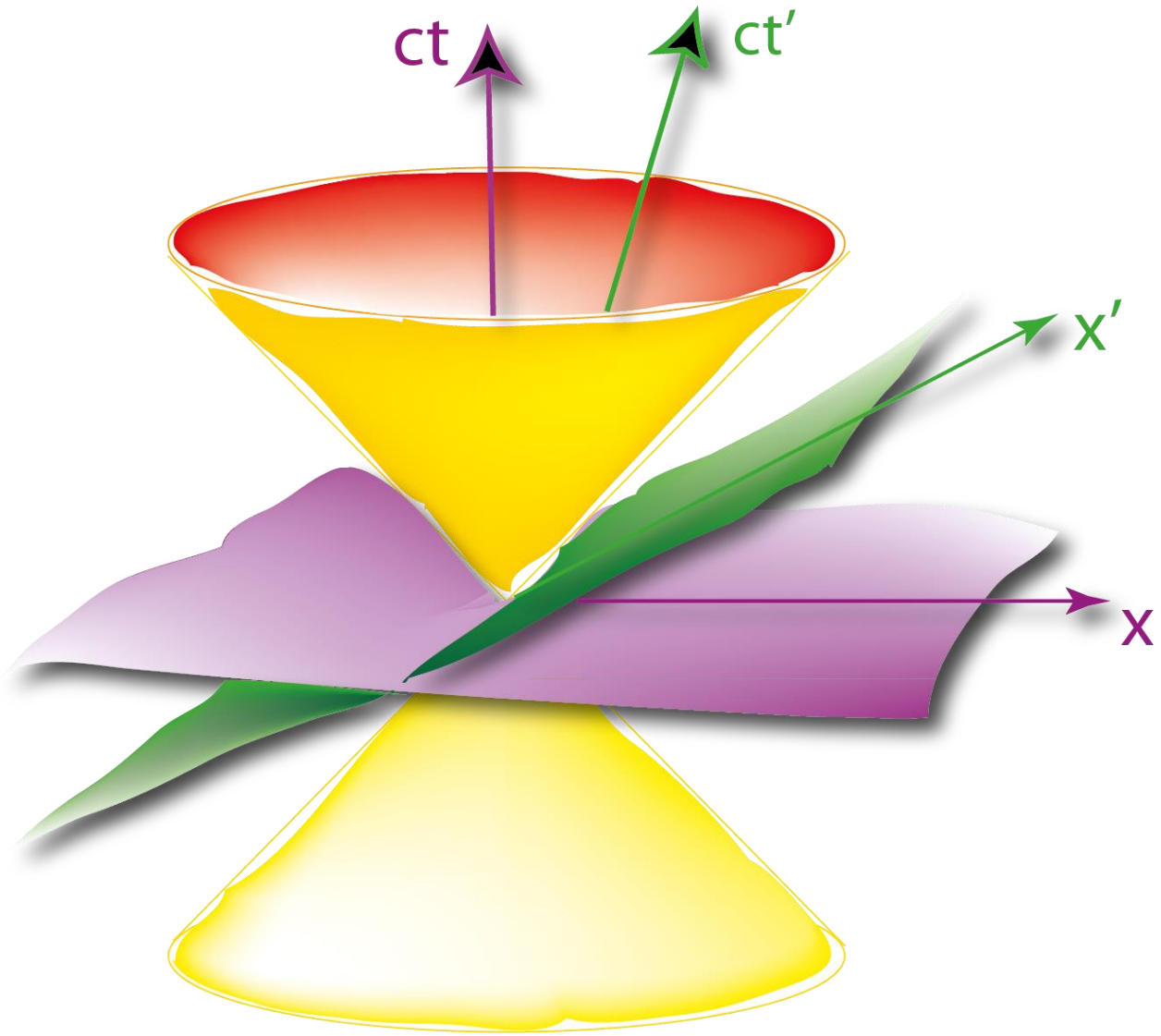
$$x' = x - Vt\,, \quad y' = y\,, \quad z' = z\,,$$

$t' = t$ Absolute Time

Composition of velocities $u' = u - V$

Link 3
Link 4

Lorentz transformations and Minkowski Spacetime



$$ct' = \frac{ct - \frac{V}{c}x}{\sqrt{1 - \frac{V^2}{c^2}}},$$

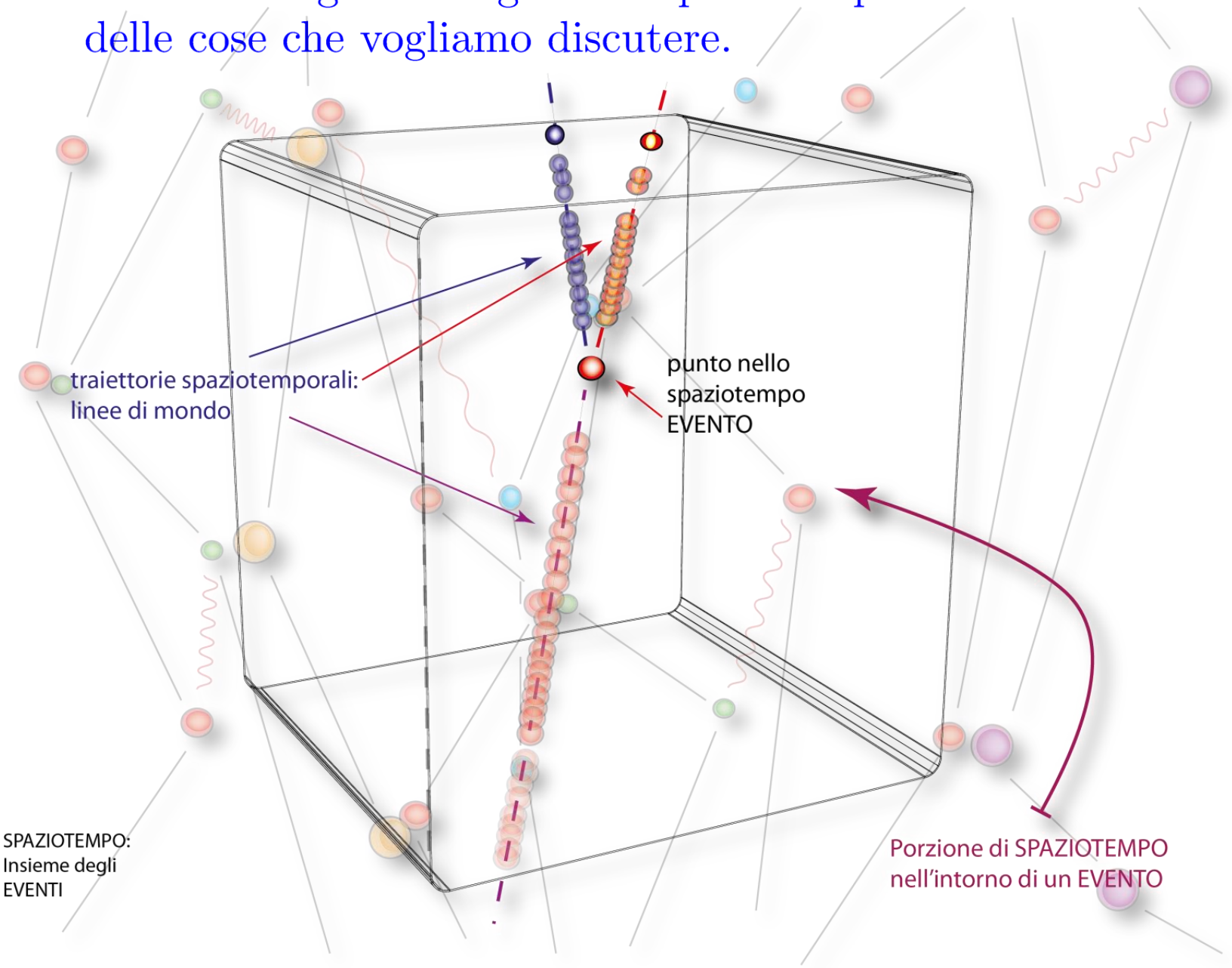
$$x' = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}},$$

$$y' = y,$$

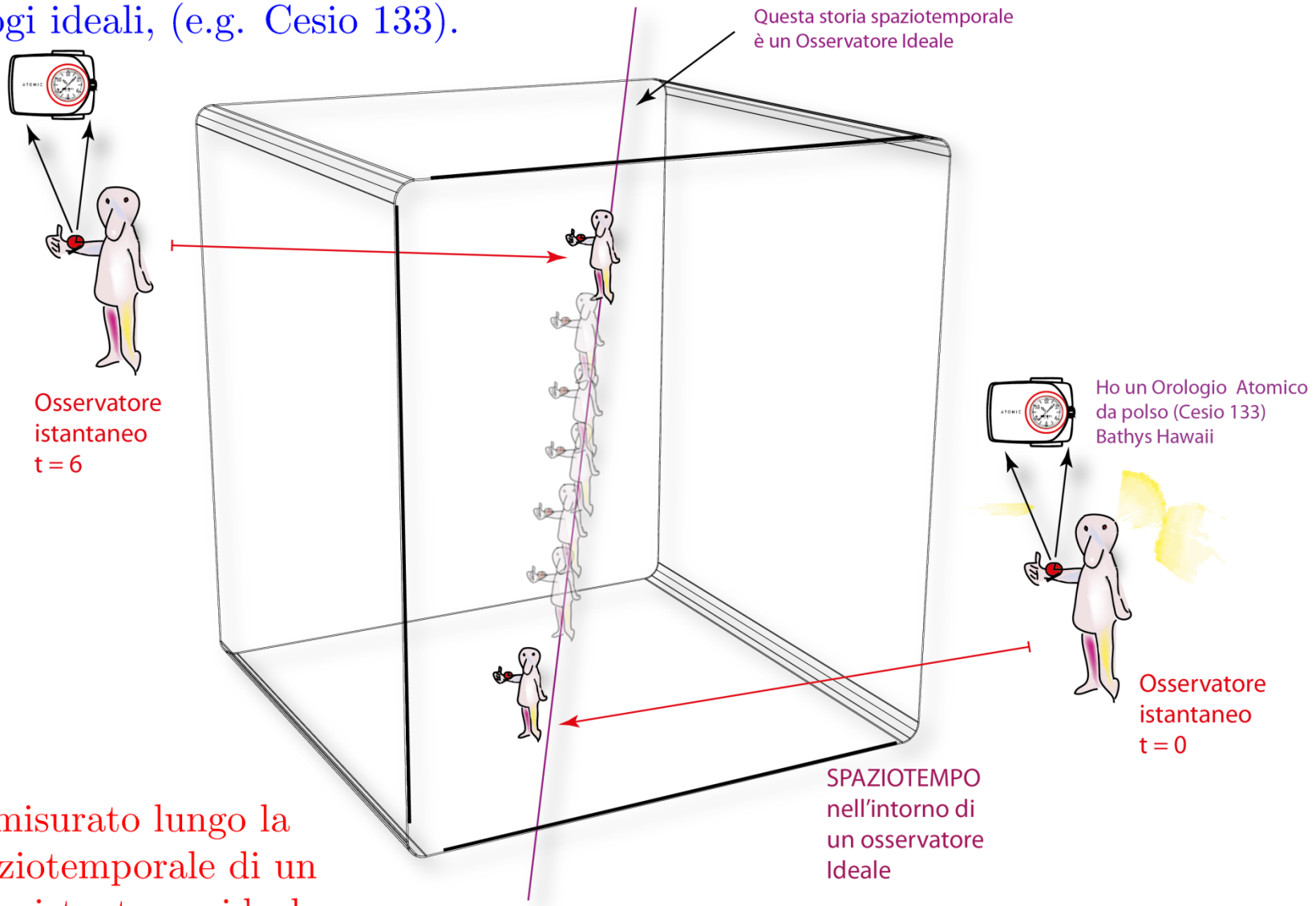
$$z' = z,$$

$$U' = \frac{U - V}{1 - \frac{UV}{c^2}},$$

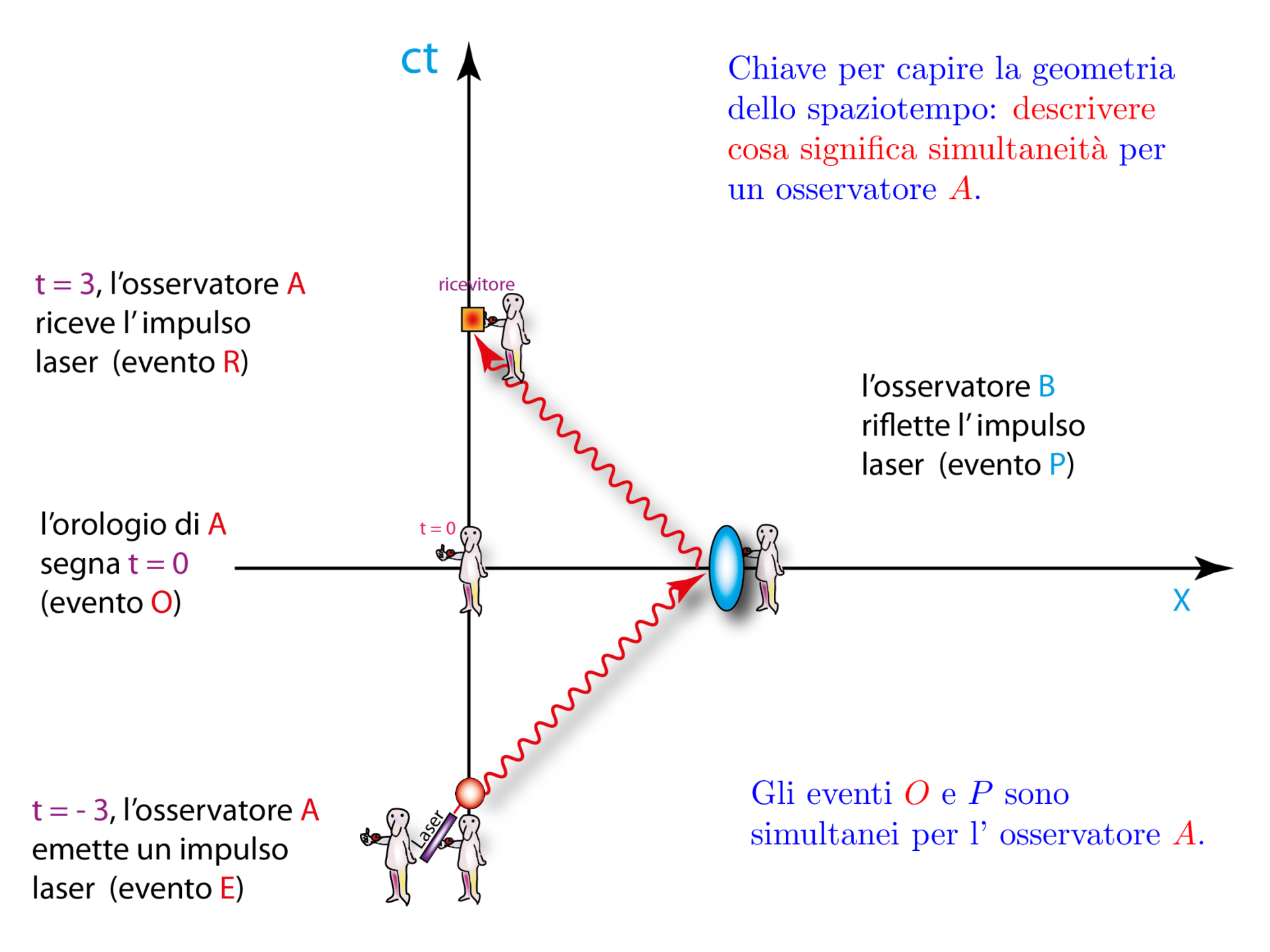
E' utile disegnare diagrammi spaziotemporali delle cose che vogliamo discutere.

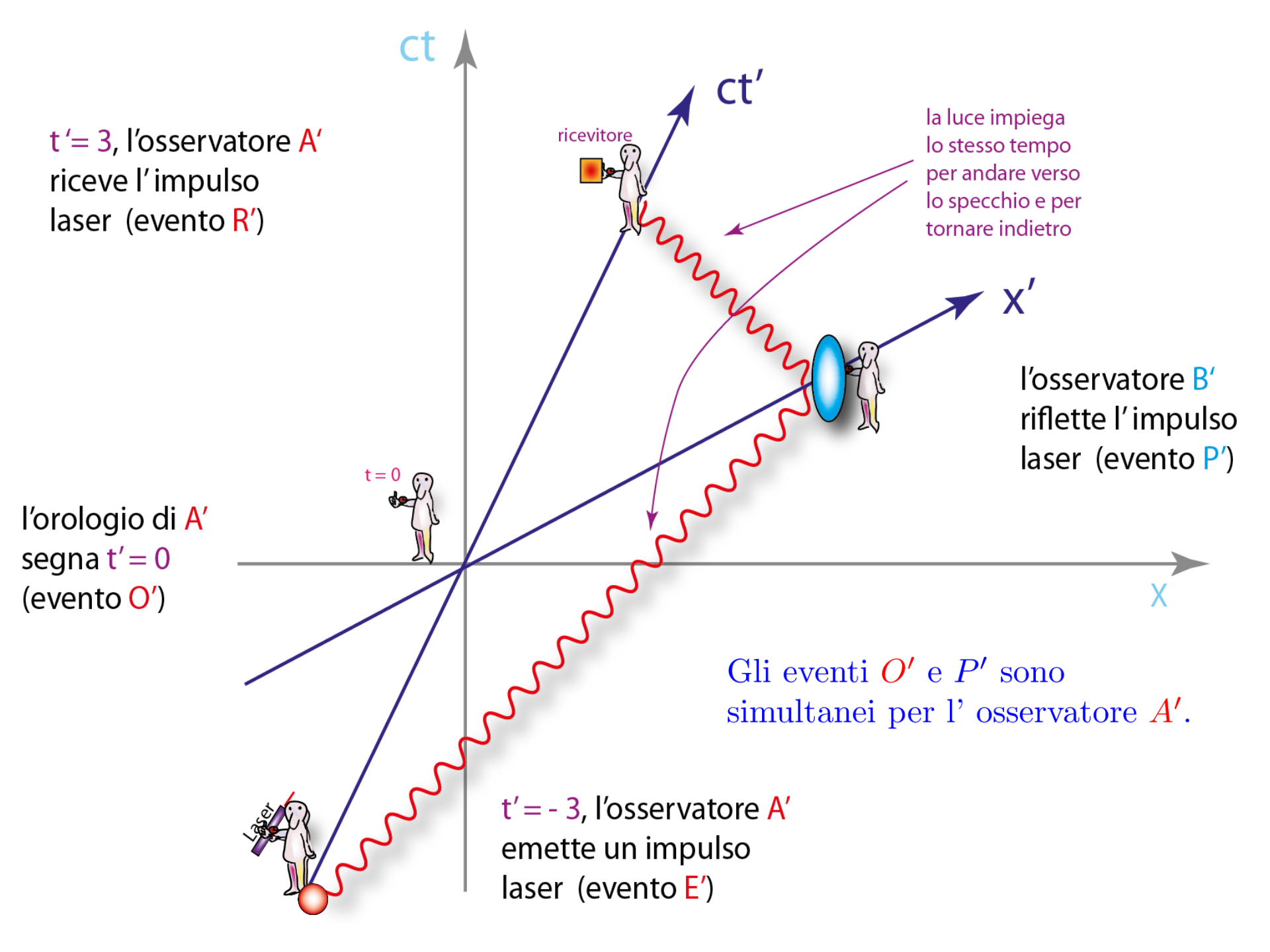


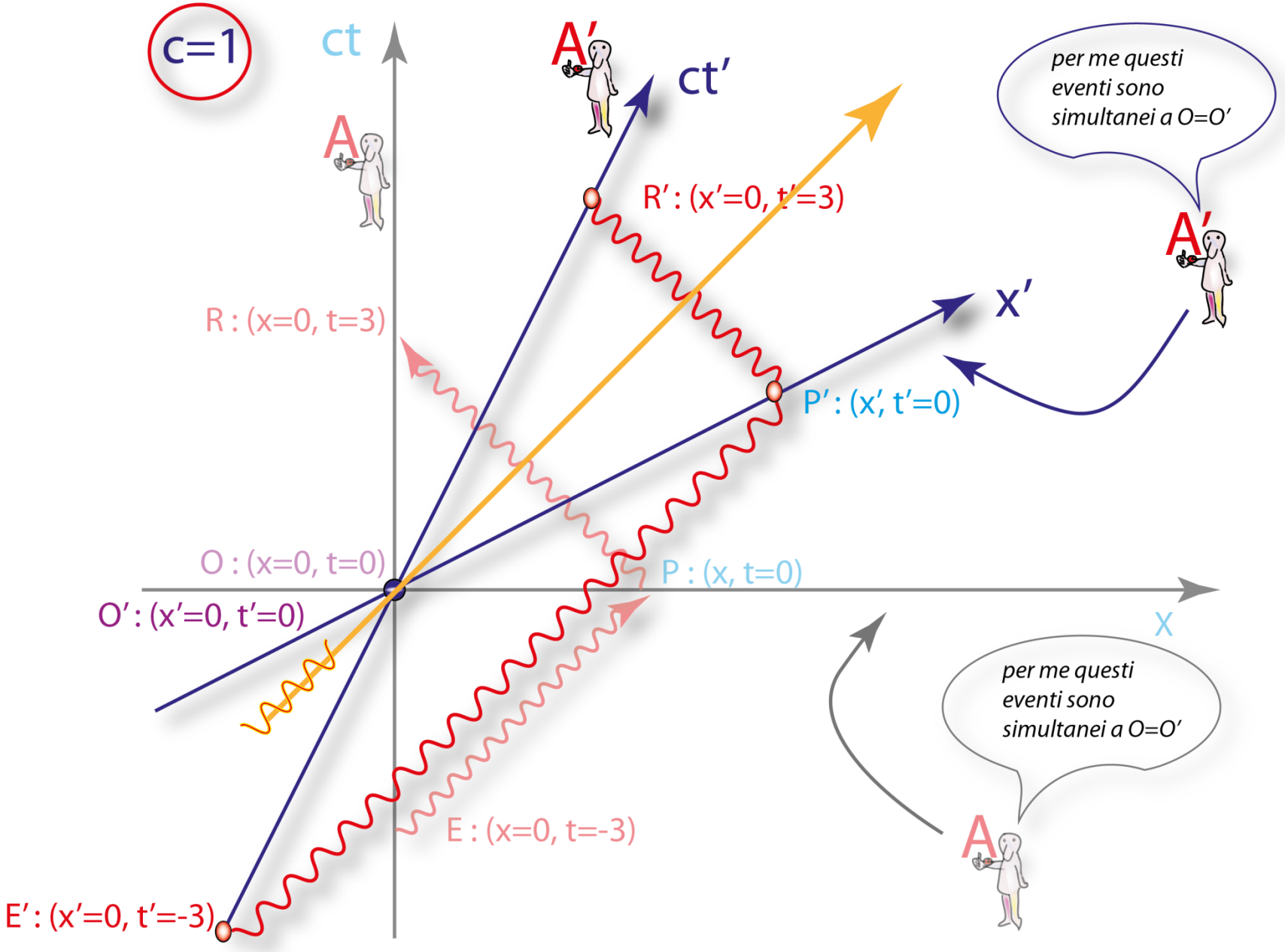
Per fare misure nello spaziotempo abbiamo bisogno di osservatori dotati di orologi ideali, (e.g. Cesio 133).

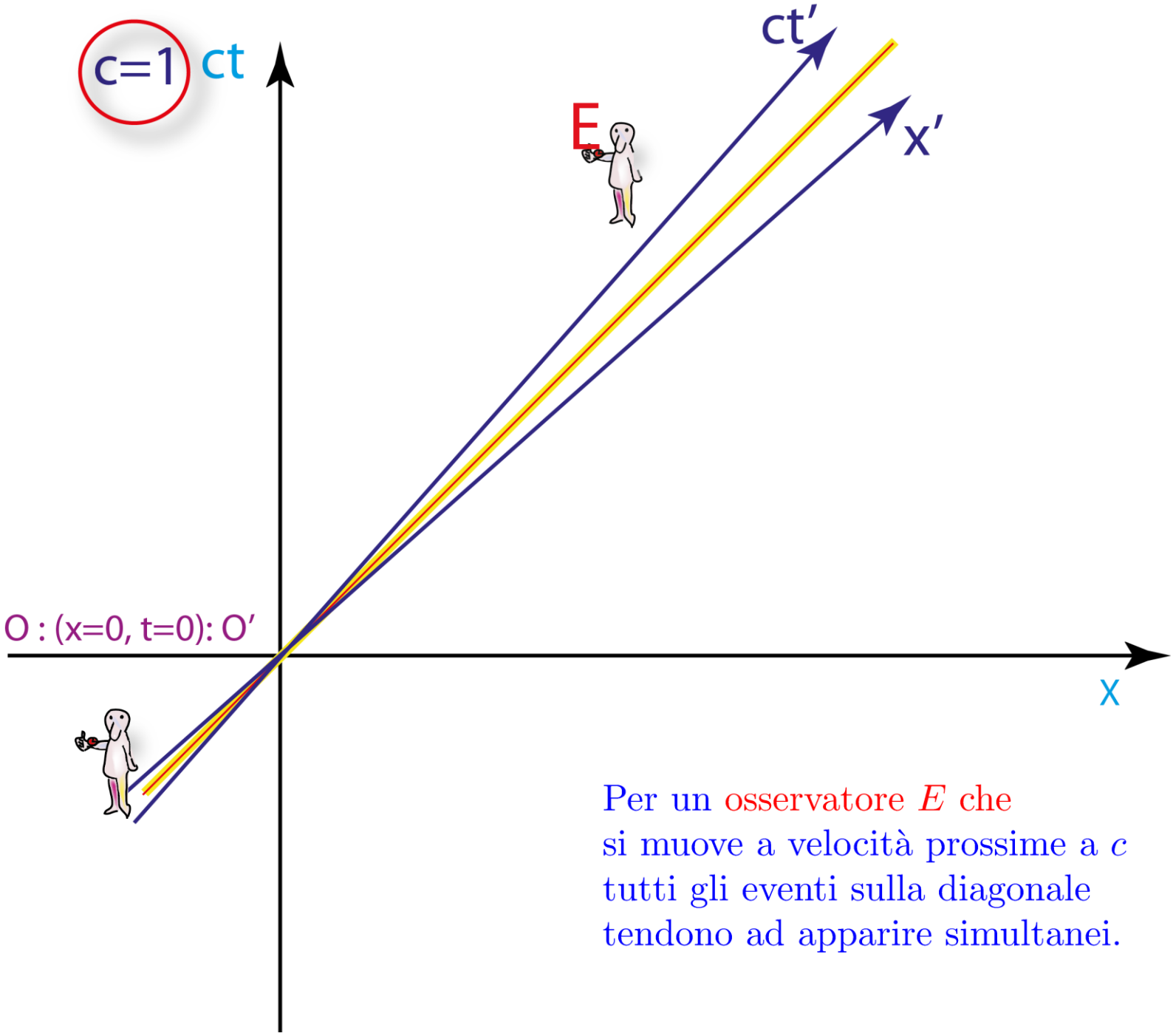


Il tempo misurato lungo la storia spaziotemporale di un osservatore istantaneo ideale misura il TEMPO PROPRIO dell'osservatore

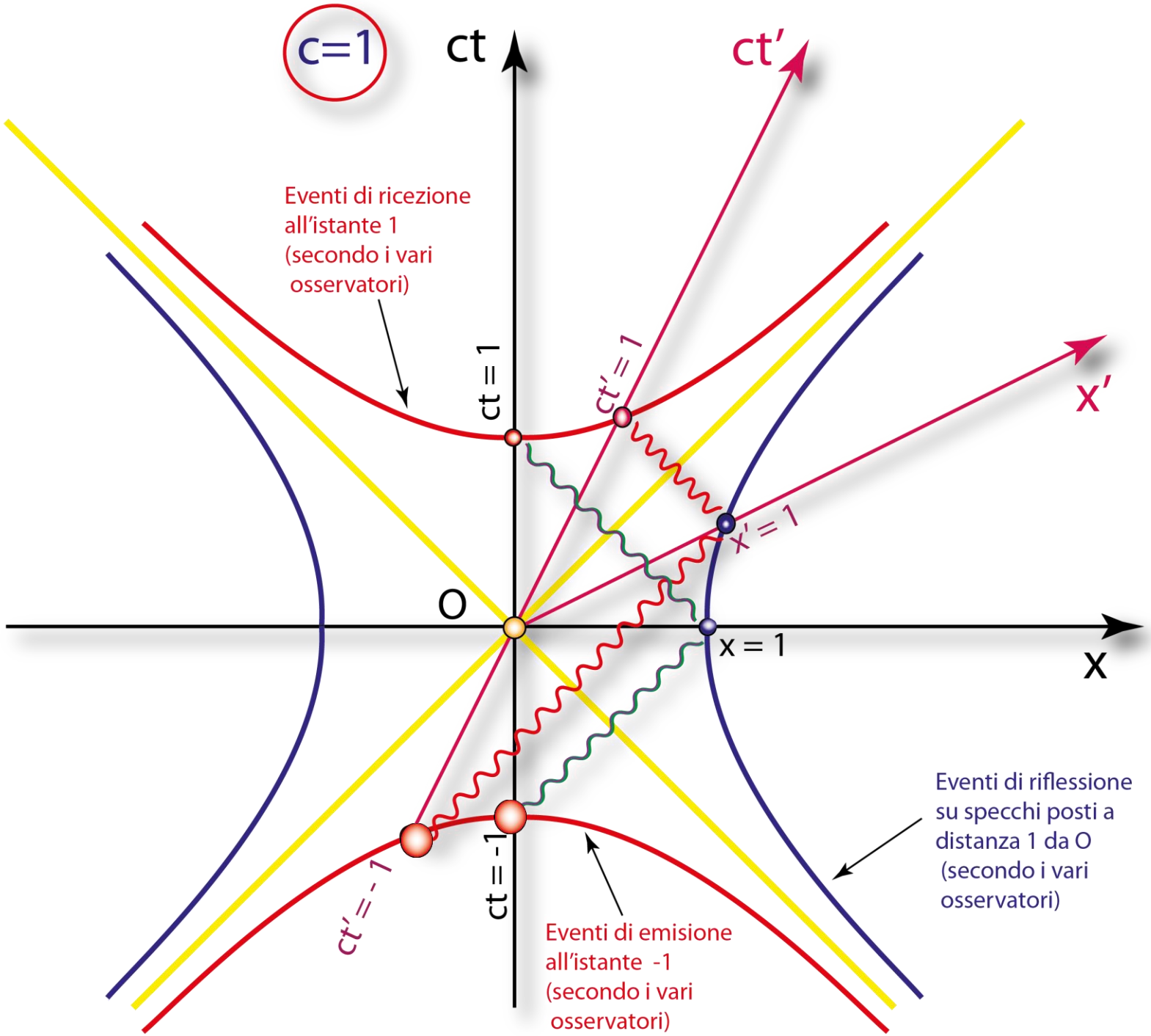

ct
Chiave per capire la geometria dello spaziotempo: descrivere cosa significa simultaneità per un osservatore A.
t = 3, l'osservatore A riceve l'impulso laser (evento R)
ricevitore
l'osservatore B riflette l'impulso laser (evento P)
l'orologio di A segna t = 0 (evento O)
t = 0
X
t = - 3, l'osservatore A emette un impulso laser (evento E)
Laser
Gli eventi O e P sono simultanei per l' osservatore A.


ct
ct′
x′
X
ricevitore
la luce impiega
lo stesso tempo
per andare verso
lo specchio e per
tornare indietro
t ′= 3, l'osservatore A′
riceve l' impulso
laser (evento R′)
l'osservatore B′
riflette l' impulso
laser (evento P′)
t = 0
l'orologio di A′
segna t′ = 0
(evento O′)
Laser
Gli eventi O' e P' sono
simultanei per l' osservatore A'.
t′ = - 3, l'osservatore A′
emette un impulso
laser (evento E′)

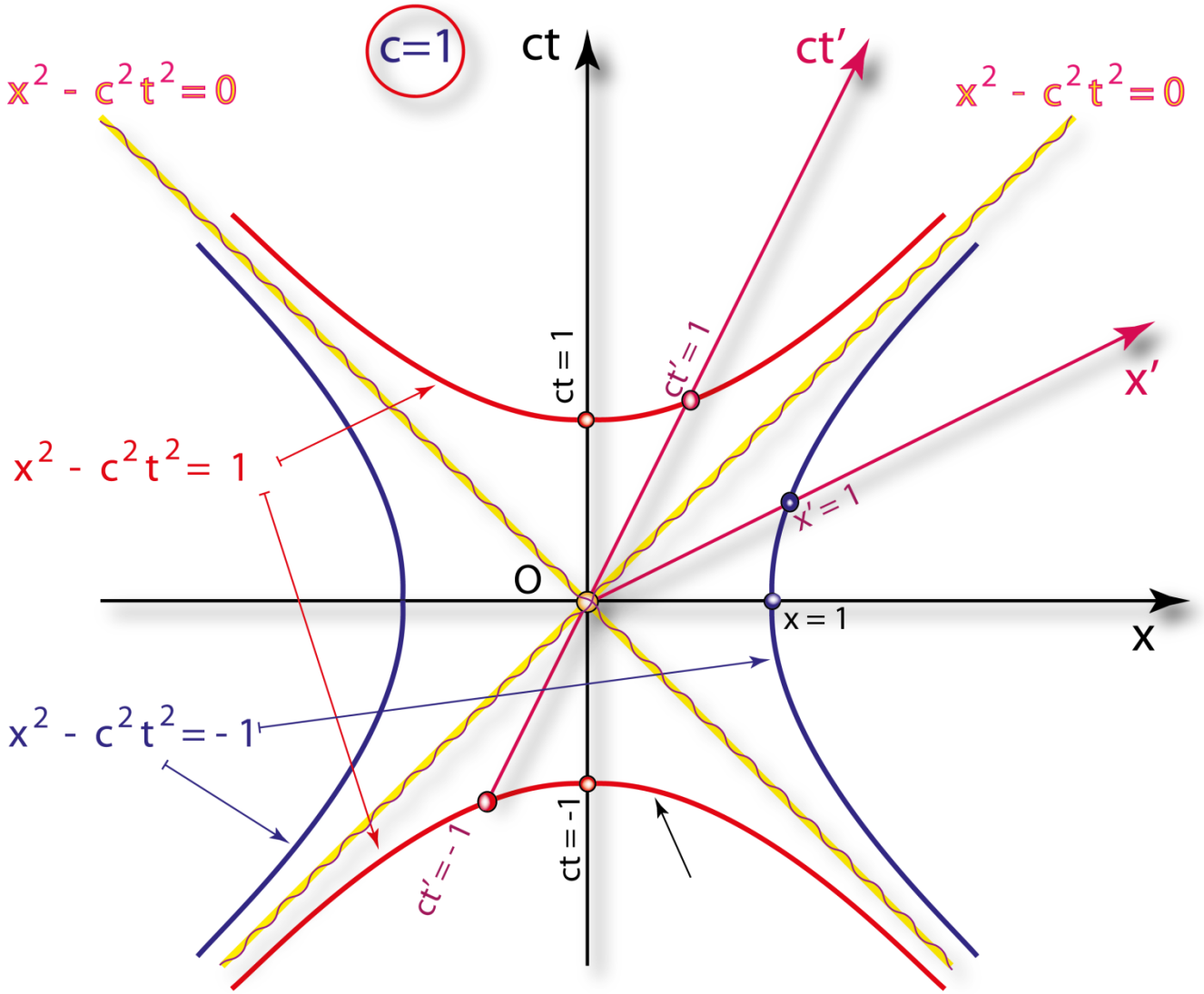

c=1
ct
ct'
A'
A
R' : (x'=0, t'=3)
R : (x=0, t=3)
x'
P' : (x', t'=0)
O : (x=0, t=0)
P : (x, t=0)
O' : (x'=0, t'=0)
X
E : (x=0, t=-3)
E' : (x'=0, t'=-3)
per me questi eventi sono simultanei a O=O'
A'
per me questi eventi sono simultanei a O=O'
A



Per un osservatore E che
si muove a velocità prossime a c
tutti gli eventi sulla diagonale
tendono ad apparire simultanei.


c=1
ct
ct′
Eventi di ricezione
all'istante 1
(secondo i vari
osservatori)
ct = 1
ct′ = 1
x′
x′ = 1
O
x = 1
x
Eventi di riflessione
su specchi posti a
distanza 1 da O
(secondo i vari
osservatori)
ct′ = - 1
ct = -1
Eventi di emisione
all'istante -1
(secondo i vari
osservatori)

Queste proprietà geometriche
che ci permettono di fare misure
nello spaziotempo sono descritte da
alcune semplici relazioni fra
le variabili x e ct:



Proprietà delle trasformazioni di Lorentz

- Relatività della simultaneità: due eventi distinti $E_1 = (x'_1, t'_1 = t')$ e $E_2 = (x'_2, t'_2 = t')$, che avvengono simultaneamente quando osservati in un riferimento inerziale R', in moto rispetto a R con velocità v, non risultano più simultanei quando osservati in R:

$$t_1 = \frac{t' + \frac{v}{c^2}x'_1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t_2 = \frac{t' + \frac{v}{c^2}x'_2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

.

- La dilatazione dei tempi: Un fenomeno che nel riferimento R' dura $\Delta t'_0 = t'_2 - t'_1$ ha, se visto nel riferimento R, durata

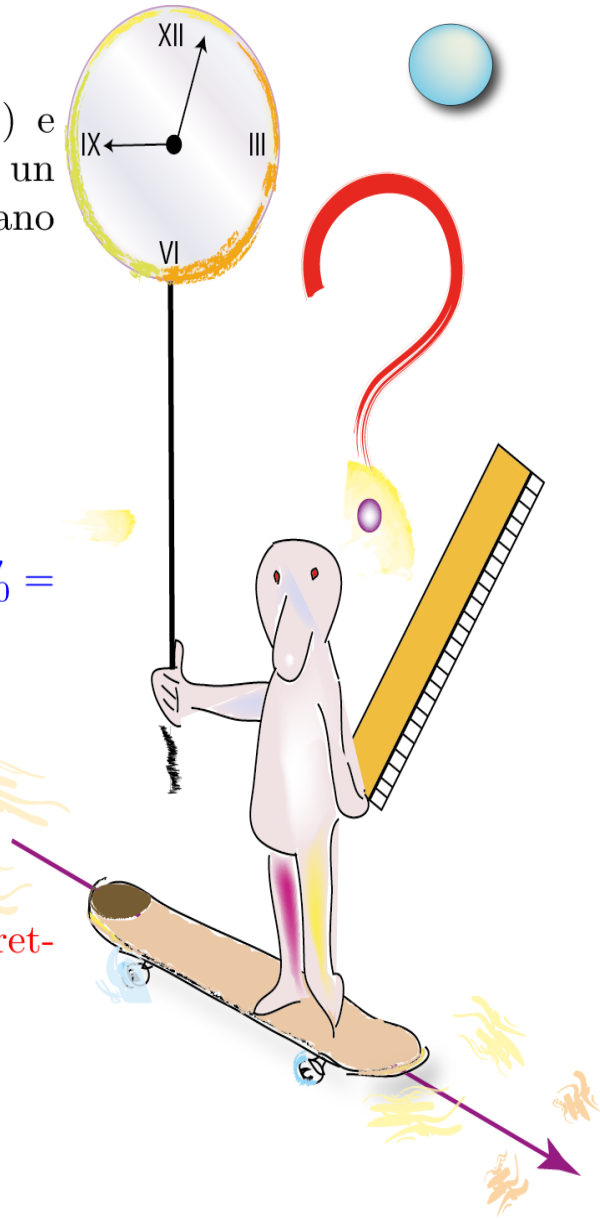
$$\Delta t = \frac{\Delta t'_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

.

- La contrazione delle lunghezze: un regolo di lunghezza $\Delta x'_0$ in moto rettilineo uniforme rispetto a un R risulta essere lungo in R

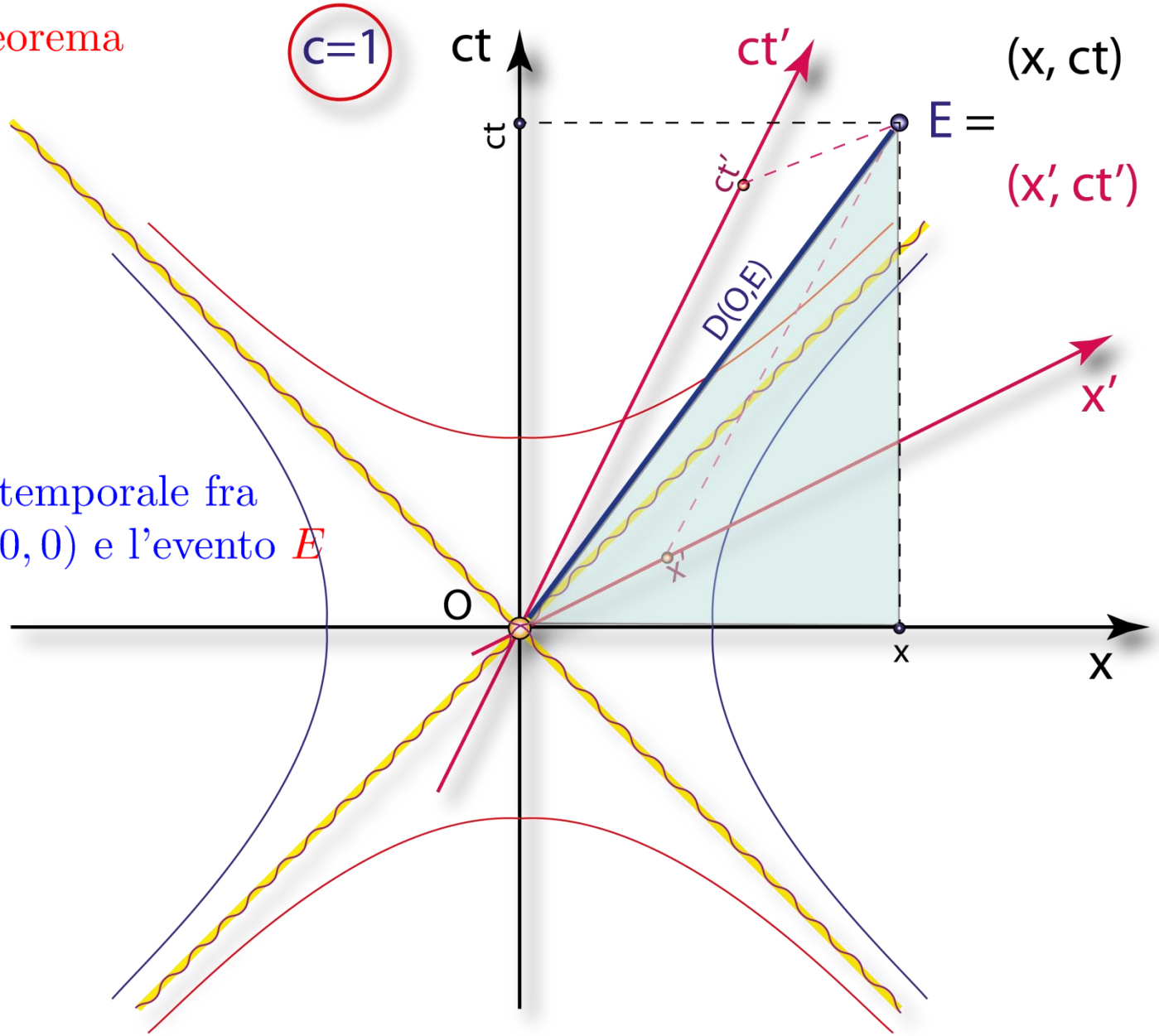
$$\Delta x = \Delta x'_0 \sqrt{1 - \frac{v^2}{c^2}}$$

.



Misure spaziali e misure temporali dipendono quindi dall'osservatore!

Un quasi-teorema di Pitagora



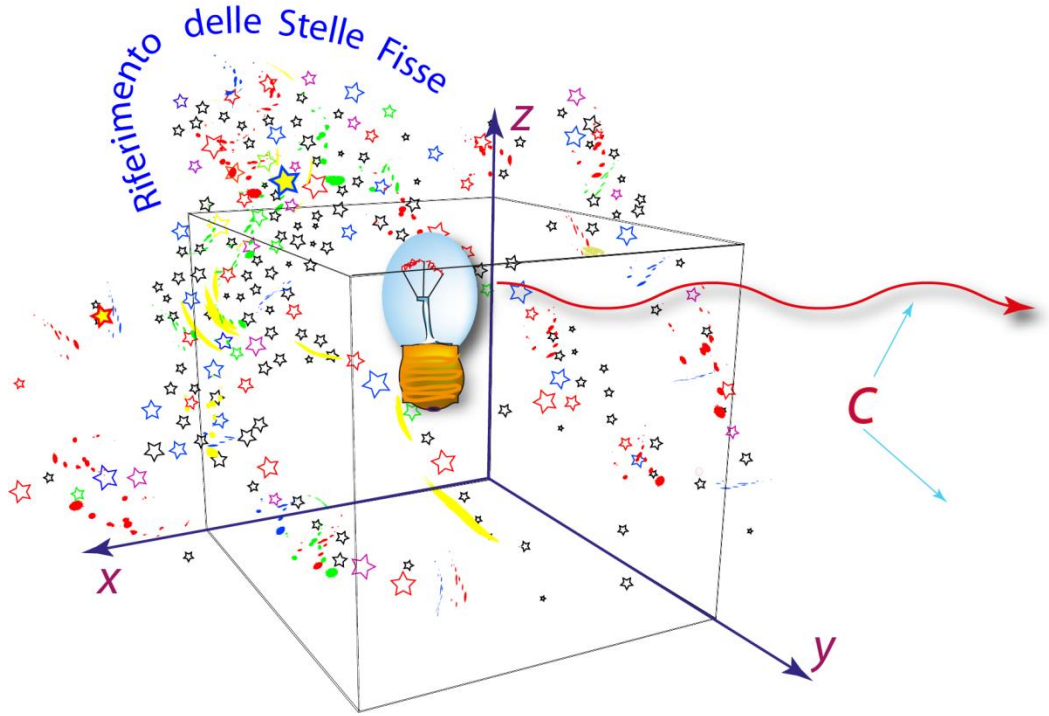
Divario spaziotemporale fra l'evento $O = (0, 0)$ e l'evento E

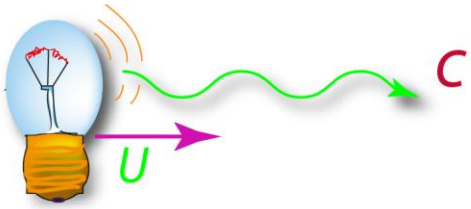
$$D(O,E) := x^2 - c^2\,t^2 = x'^2 - c^2\,t'^2$$

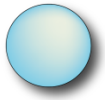
Link 4

Link 5  The speed of light in vacuum $c = 2.99792458 \times 10^{10}\, cm/sec$

- The speed of light in vacuum c does not depend from the velocity of the emitting source.
- This contrasts with Galilean Relativity:
- According to Galilean velocity composition we should have $C' = C + U$!

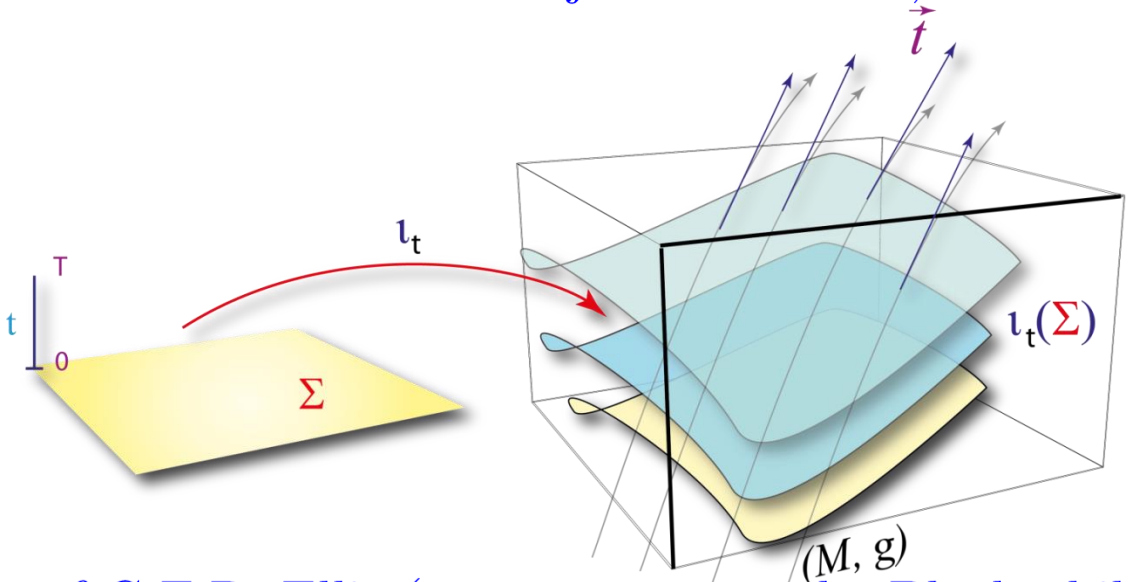




Link 6 

Comments on the Block Universe interpretation

General Relativity is a deterministic theory. Roughly speaking, given suitable data at a given "instant", Einstein's equation allow to generate spacetime as a dynamical evolution of these data. A strict (somehow, even naive) interpretation of this and of the global nature of the known (few) exact spacetimes solution of Einstein's equations (Minkowski, Schwarzschild, Kerr, Friedmann-Lemaitre-Robertson-Walker spacetimes, ...) gave rise to the idea of a *block universe*: the universe just is, as a fixed block! Time is just an illusion, it does not "roll on".

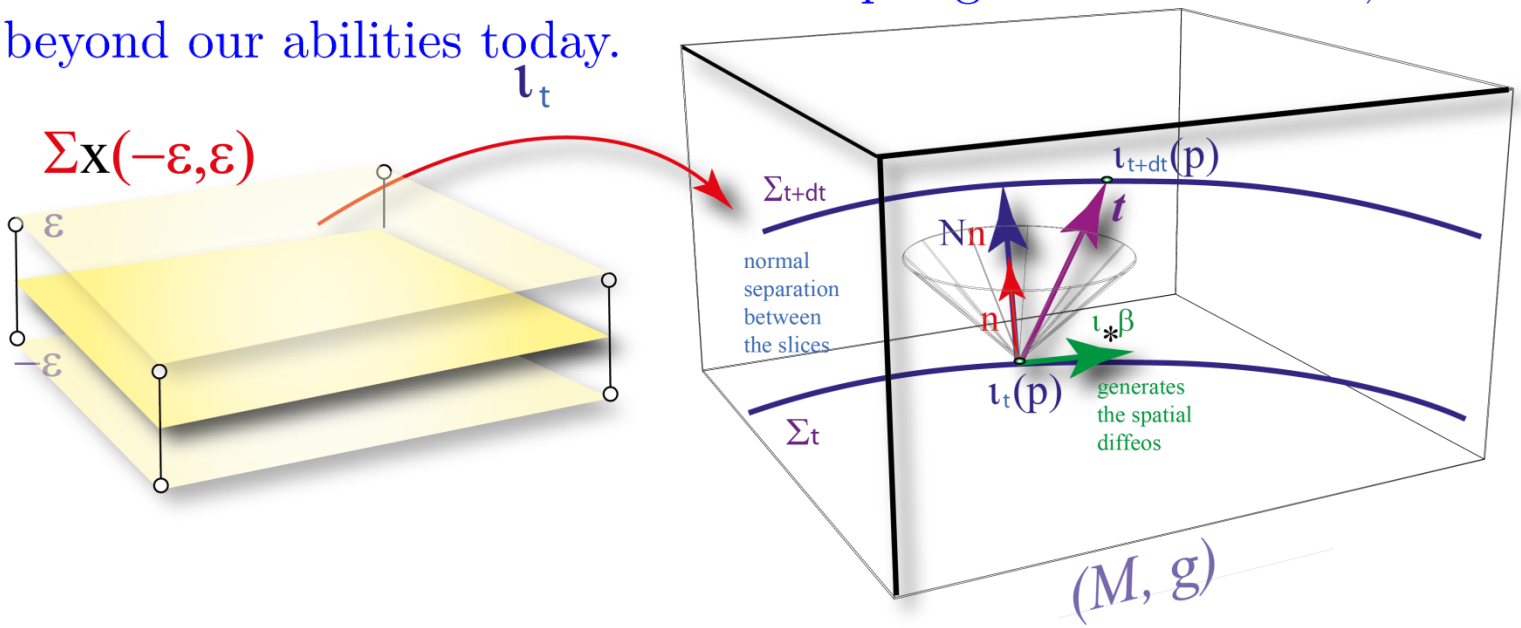


To use the words of G.F.R. Ellis (an opponent to the Block philosophy), *space and time are represented as merged into an unchanging spacetime entity, with no particular sections identified as the present and no evolution of spacetime taking place*

I do not consider the Block Universe interpretation correct since:

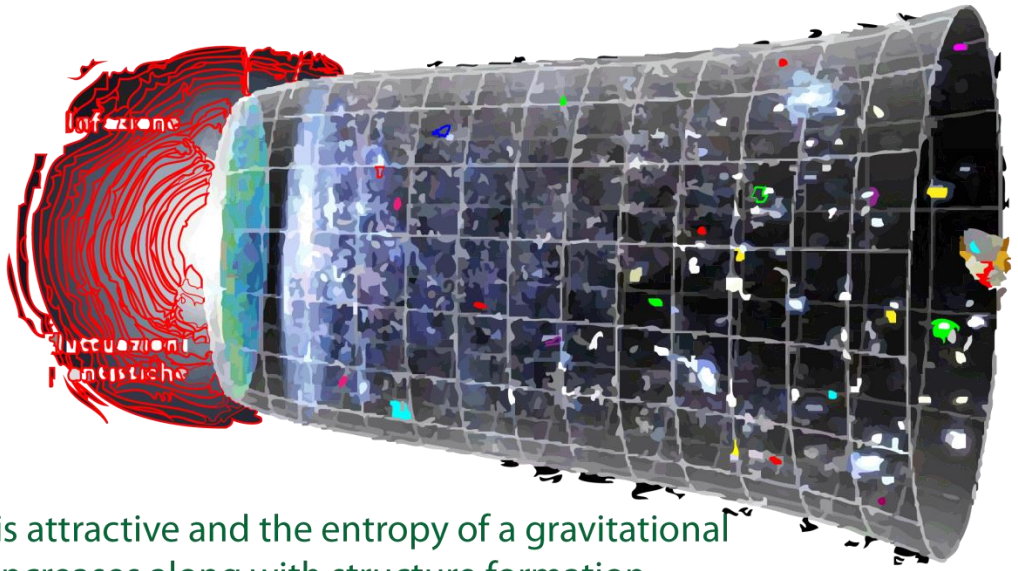
- It abuses the nature of the Cauchy problem in general relativity: this is a highly non-linear theory, and the deterministic propagation of initial data can be typically controlled just for small time. Globality is under control only under very special circumstances, and a *local* Block universe, which is what we can really get with the rigorous analysis of Einstein equations, is a nonsense.

- It does not take into account that singularities develops in finite proper time; it is not yet clear if these are hidden by event horizons (cosmic censorship), and even if one could dream of having mathematical control of the evolution of a moltitude of sources collapsing into black holes, this is totally beyond our abilities today.



- The most important objection to the block interpretation of the dynamics of spacetime, resides, in my opinion, in the fact that for a complex gravitating system, such as that associated to a cosmological spacetime, there will be sensible dependence from (even local) initial data. This gives rise to the onset of a complex relaxational behavior (gravity is attractive) depending on (local) spacetime regions (*i.e.* over observer's proper time scales and spatial averaging regions). Thermodynamic irreversibility and an arrow of time dominate. (Space)Time will unfold.

The Arrow of Time as an emergent property of the cosmological dynamics



Gravity is attractive and the entropy of a gravitational system increases along with structure formation.
Do Black Holes, endpoint of gravitational structures, with their immense entropy, play a role in the "Physics" of Time: ... Time is frozen (as seen from infinity) near the Event Horizon ...

Link 6

A Few References

- P. C. W. Davies, *That Mysterious Flow*, Scientific American 287, pp.40 (2002)
- G. F. R. Ellis, *Physics in the Real Universe: Time and Spacetime*, Gen.Rel.Grav. 38 (2006) 1797-1824, arXiv:gr-qc/0605049
- S.W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge Univ. Press (1973)
- T. Jacobs and C. Maes, *Reversibility and Irreversibility within the Quantum Formalism*, Physicalia Mag. 27 (2005), 119-130, arXiv:quant-ph/0508041
- R. Laughlin, *A different Universe*, Basic Books, New York, (2005)
- D. H. Mellor, *Real Time II*, Routledge, London, 1998
- R. Penrose, *The Road to Reality*, Jonathan Cape London, (2004)
- H. Poincaré, *La Mesure du Temps*, Revue de Metaphysique et de Morale, T. 6, No. 1 (Janvier 1898), pp. 1-13, Presses Universitaires de France
- W. Rindler, *Relativity, Special, General, Cosmological*, Oxford Univ. Press (2001)
- S.Savitt, *Being and Becoming in Modern Physics*, The Stanford Encyclopedia of Phylosophy, (2002), E. N. Zalta ed.